Forces – Exercises (33 pages; 25/3/25)

If a block of mass m is resting on a surface, prove that the reaction force of the block on the surface is mg.

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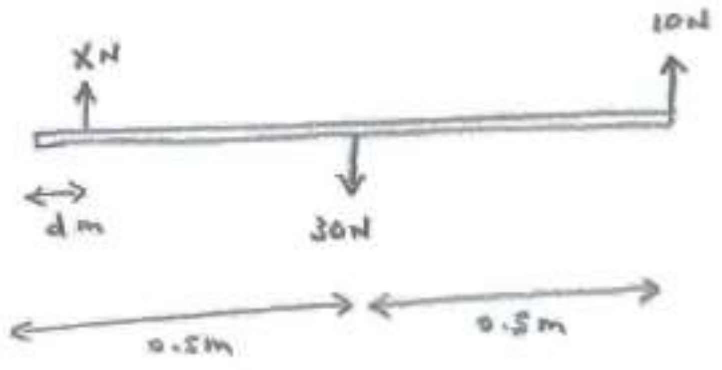
Solution

Let R be the reaction force of the block on the surface.

By N3L, the reaction force of the surface on the block is also R.

Applying N2L to the block, $mg - R = 0$, so that $R = mg$.

Vertical forces of X, 30 and 10 N are applied to a light rod of length 1 m, as shown in the diagram. The force of X N is applied at a distance of d m from the left-hand end, and the force of 30N is applied at the mid-point of the rod.



(a) What values must X and d have in order for the rod to be in equilbrium?

(b) The force of X N is removed, and the forces of 30N and 10N are to be replaced with a single force having the same effect as these two forces. What is the size and line of action of this single force?

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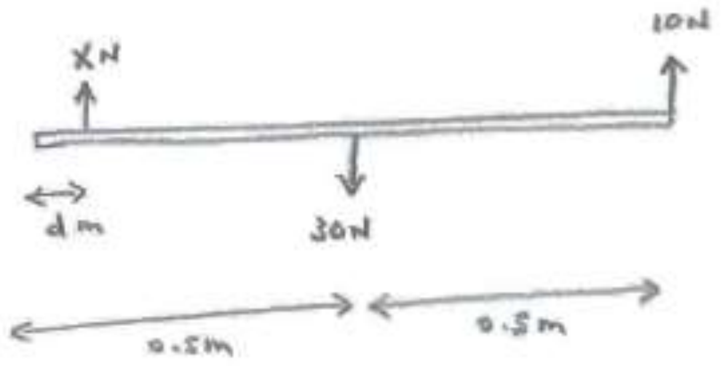

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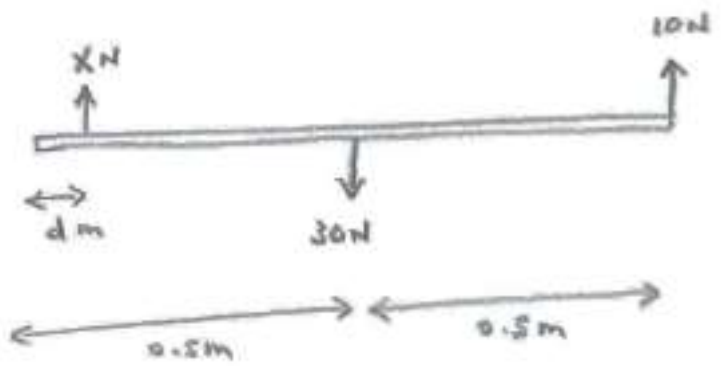
Solution



(a) Vertical equilbrium $\Rightarrow X + 10 = 30 \Rightarrow X = 20$

Taking moments about the right-hand end (for example):

$30(0.5) - 20(1-d) = 0 \Rightarrow -5 + 20d = 0 \Rightarrow d = 0.25$

[Whenever the forces are balanced, the total moment will be the same about any point; eg taking moments about the mid-point instead:

$10(0.5) - 20(0.5-d) = 0 \Rightarrow -5 + 20d = 0$, as before

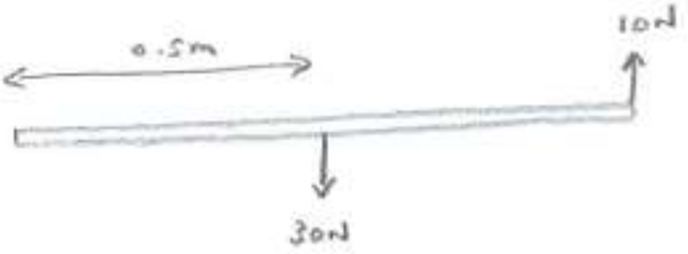
or, about the point where X is applied:

$10(1-d) - 30(0.5-d) = 0 \Rightarrow -5 + 20d = 0$]

(b) From (a), X counteracts the effect of the other two forces to give equilibrium.

Now a force of 20 N acting at the same position as X, but in the opposite direction, will also be counteracted by X. Hence it follows that this force is equivalent to the forces of 30 N and 10 N.

Thus the two systems shown below are equivalent.



Alternative method:

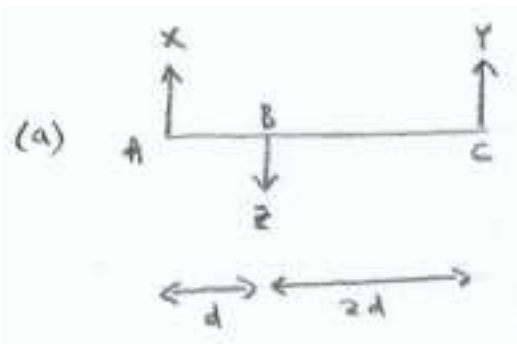
The single equivalent force must be of magnitude 20 N, acting downwards (this being the net effect of the two forces). Suppose that it acts at a distance d from the left-hand end.

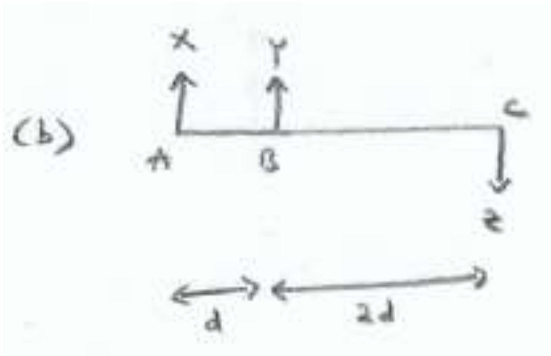
Then we require the moment of this force about the left-hand end (say) to equal the net moment of the two forces.

So $-20d = 10(1) - 30(0.5) = -5 \Rightarrow d = 0.25$

(Note: the single force could not act beyond the left-hand end, as that would give rise to a positive moment, which could not be equated to -5)

(i) Which of the following systems of forces could be in equilbrium? (with X, Y and Z > 0)





(ii) Assuming that $X + Y = Z$, show that the total moments about A, B and C are equal, in both of the cases in (i).

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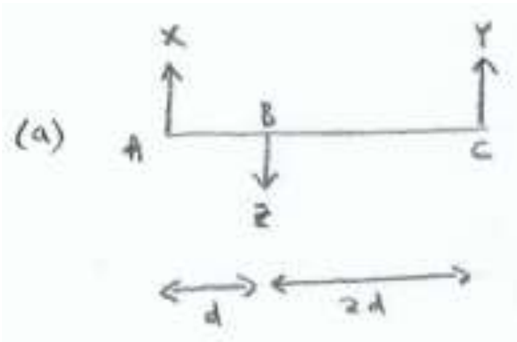
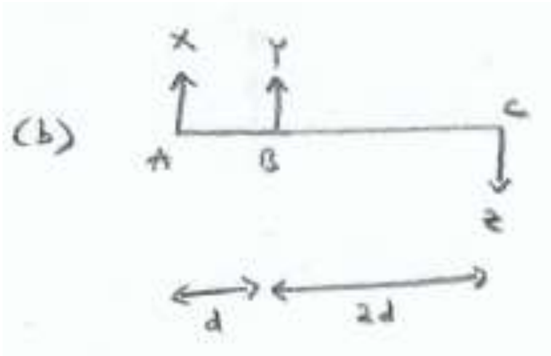




(ii) Assuming that $X + Y = Z$, show that the total moments about A, B and C are equal, in both of the cases in (i).

Solution

(i) (a) Vertical equilibrium requires that $X + Y = Z$.

For rotational equilibrium, taking moments about B, $2dY - dX = 0$, so that $X = 2Y$.

Thus there is equilibrium provided that $Y = \frac{X}{2}$ and $Z = \frac{3X}{2}$.

[Note: As about to be shown in (ii), we can take moments about any point, provided that $X + Y = Z$]

(b) If we take moments about B, we obtain $-dX - 2dZ$, which cannot equal zero. Thus the system cannot be in equilibrium.

[With 3 forces, the directions of the forces must alternate for equilibrium to be possible.]

(ii) (a) $M(A): 3dY - dZ = 3dY - d(X + Y) = d(2Y - X)$

$M(B)\colon 2dY - dX = d(2Y - X)$

$M(C)\colon 2dZ - 3dX = 2d(X + Y) - 3dX = d(2Y - X)$

(b) $M(A)\colon dY - 3dZ = dY - 3d(X + Y) = -d(3X + 2Y)$

$M(B)\colon - dX - 2dZ = -dX - 2d(X + Y) = -d(3X + 2Y)$

$M(C)\colon - 2dY - 3dX = -d(3X + 2Y)$

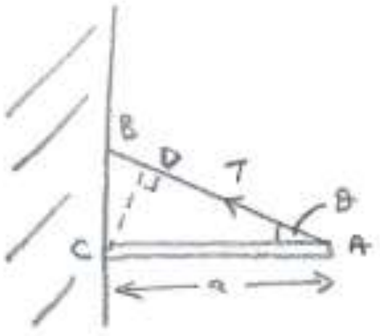
[Thus the total moment will be the same about any point, provided that the forces balance; regardless of whether there is rotational equilibrium.]

Show that the moment of T about C is the same:

(i) if T is multiplied by CD

(ii) T is resolved into horizontal & vertical components at A

(iii) T is resolved into horizontal & vertical components at B



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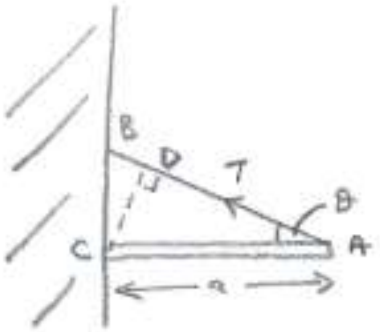
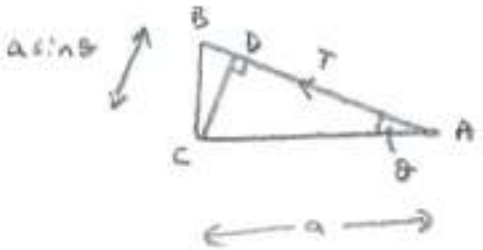


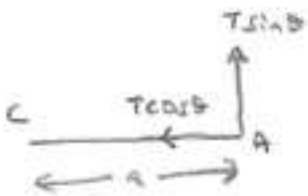
Solution

(i)



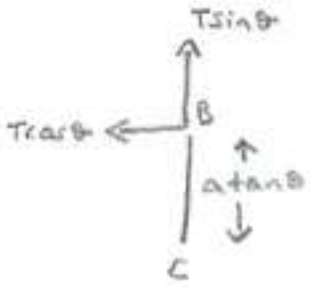
moment $= T \times CD = Tasin\theta$

(ii)



moment $= (Tcos\theta)(0) + (Tsin\theta)a = Tasin\theta$

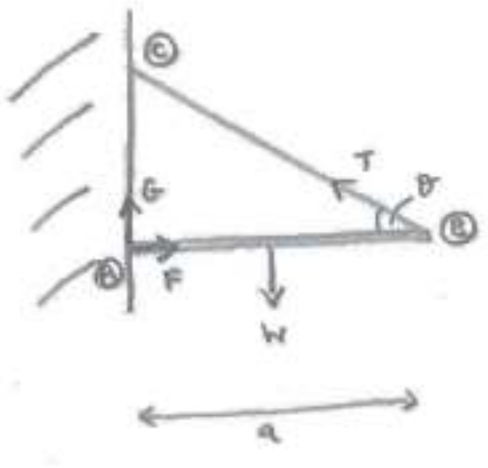
(iii)



Referring to the original diagram, $CB = atan\theta$, so that

moment = $(Tcos\theta)(atan\theta) + (Tsin\theta)(0) = Tasin\theta$

[Alternative Moments Methods]



A rod AB is attached to a wall at A, and held in a horizontal position by a rope BC.

Show that, as an alternative to resolving forces horizontally and vertically, and taking moments about A, it is also possible to:

(a) resolve forces horizontally and take moments about A & B,

or (b) take moments about A, B & C;

but that it is not possible to do the following:

(c) resolve forces vertically and take moments about A & B,

or (d) take moments about A, B & the midpoint of AB

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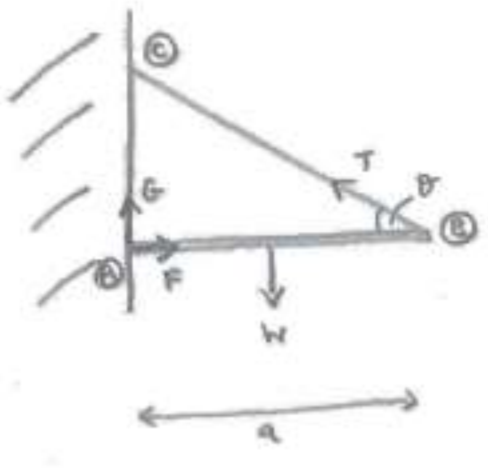

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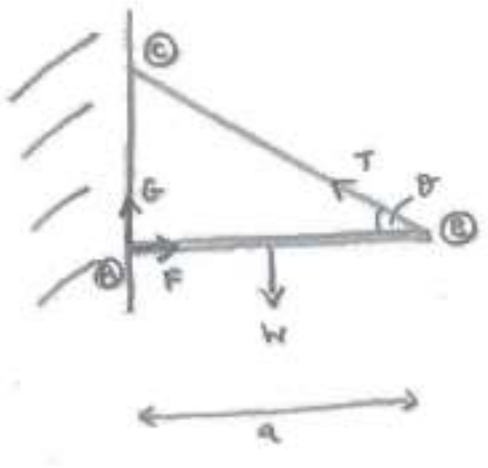

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Show that, as an alternative to resolving forces horizontally and vertically, and taking moments about A, it is also possible to:

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or (b) take moments about A, B & C;

but that it is not possible to do the following:

(c) resolve forces vertically and take moments about A & B,

or (d) take moments about A, B & the midpoint of AB

Solution

Resolving forces horizontally and vertically,

$F = Tcos\theta$ (1) & $W = G + Tsin\theta$ (2)

Taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (3)

Then (3) $\Rightarrow T = \frac{W}{2sin\theta}$

and hence (1) $\Rightarrow F = \frac{Wcot\theta}{2}$

and (2) $\Rightarrow G = W - \frac{W}{2} = \frac{W}{2}$

Following method (a) instead,

resolving horizontally gives $F = Tcos\theta$ (4);

taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (5),

and taking moments about B gives $-Ga + W\left(\frac{a}{2}\right) = 0$ (6)

Then from (6), $G = \frac{W}{2}$;

from (5), $T = \frac{W}{2sin\theta}$,

and from (4), $F = \frac{Wcot\theta}{2}$

Following method (b) instead,

taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (7);

taking moments about B gives $-Ga + W\left(\frac{a}{2}\right) = 0$ (8),

and taking moments about C gives

$F(atan\theta) - W\left(\frac{a}{2}\right) = 0$ (9)

Then from (8), $G = \frac{W}{2}$;

from (7), $T = \frac{W}{2sin\theta}$,

and from (9), $F = \frac{Wcot\theta}{2}$

However, following method (c):

resolving vertically gives $W = G + Tsin\theta$ (10);

taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (11),

and taking moments about B gives $-Ga + W\left(\frac{a}{2}\right) = 0$ (12),

but we have no equation involving F

Also, following method (d):

taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (13);

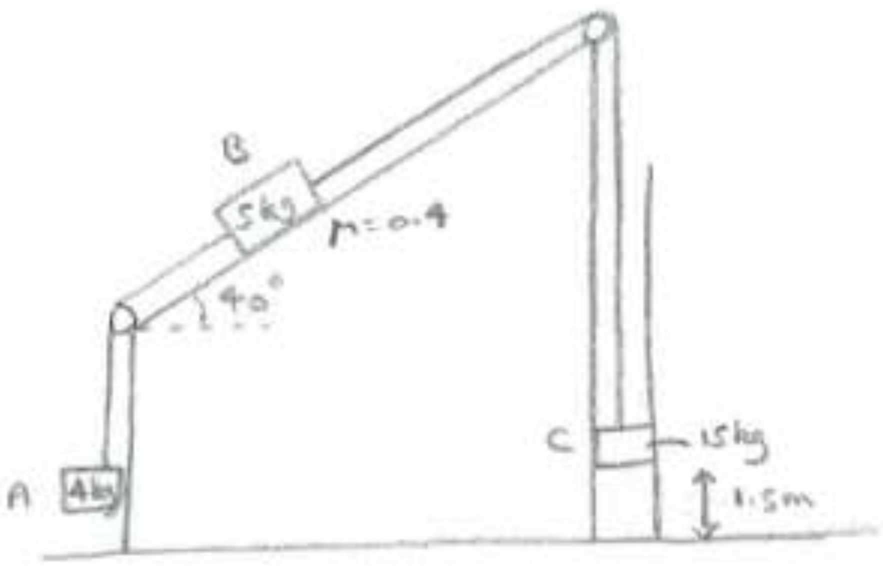
taking moments about B gives $-Ga + W\left(\frac{a}{2}\right) = 0$ (14),

and taking moments about the midpoint of AB gives

$-G\left(\frac{a}{2}\right) + (Tsin\theta)\left(\frac{a}{2}\right) = 0$,

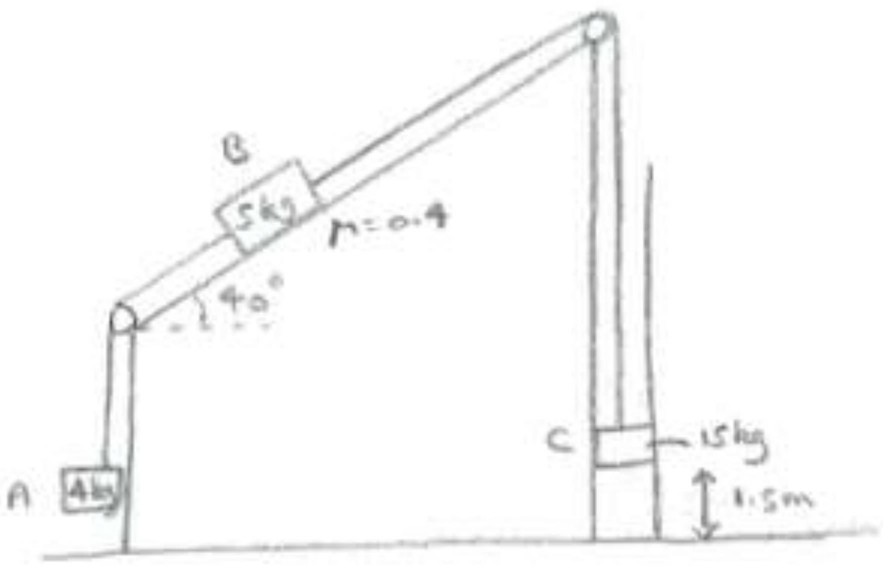
and once again we have no equation involving F

Referring to the diagram below, A has mass $4kg$ and hangs freely, B has mass $5kg$ and moves on a rough slope which makes an angle 40° with the horizontal, with a coefficient of friction of 0.4, and C has mass $15kg$ and slides in a groove, against a constant frictional force of $15N$. The objects are connected by light inextensible ropes, which pass over smooth pegs. Initially C is $1.5m$ above the ground.



The system is released from rest. Find how far up the slope B travels. It can be assumed that A and B don't reach the next peg, and that the rope connecting A and B remains taut. Also, air resistance can be ignored.

Referring to the diagram below, A has mass $4kg$ and hangs freely, B has mass $5kg$ and moves on a rough slope which makes an angle $40°$ with the horizontal, with a coefficient of friction of 0.4, and C has mass $15kg$ and slides in a groove, against a constant frictional force of $15N$. The objects are connected by light inextensible ropes, which pass over smooth pegs. Initially C is $1.5m$ above the ground.



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Solution

Method 1: N2L & suvat approach

Until C hits the ground:

Applying N2L to A: $T_{AB} - 4g = 4a$ (1)

Applying N2L to B:

$T_{BC} - T_{AB} - 5gsin40° - \mu(5gcos40°) = 5a$ (2)

Applying N2L to C: $15g - T_{BC} - 15 = 15a$ (3)

Then $(1) + (2) + (3)$ gives

$-4g - 5gsin40° - (0.4)(5gcos40°) + 15g - 15 = 24a$,

so that $a = \frac{9.8}{24}(11 - 5sin40° - 2cos40°) - \frac{15}{24} = 1.92871ms^{-2}$

Speed of B after travelling $1.5m$:

$'v^2 = u^2 + 2as' \Rightarrow v^2 = 2(1.92871)(1.5) = 5.78613$

$\Rightarrow v = 2.40544ms^{-1}$ (*)

Once C hits the ground, the rope BC becomes slack, and the N2L eq'ns become:

$A: 4g - T_{AB}{}' = 4a'$ $(1')$

(where a' is now the acceleration towards the ground)

$B: T'_{AB} + 5gsin40° + \mu(5gcos40°) = 5a'$ $(2')$

(noting that friction is still acting to oppose motion up the slope)

Then $(1')$+$(2')$ gives

$4g + 5gsin40° + \mu(5gcos40°) = 9a'$,

so that $a' = \frac{9.8}{9}(4 + 5sin40° + (0.4)5cos40°) = 9.52345ms^{-2}$

Then, from (*), $'v^2 = u^2 + 2as' \Rightarrow 0 = 2.40544^2 + 2(-9.52345)s$,

so that the total distance moved up the slope is

$1.5 + \frac{2.40544^2}{2(9.52345)} = 1.80378 = 1.80m\ (3sf)$

Method 2: Energy approach

[KE is dissipated by C on hitting the ground, and in order to determine this KE the tension in the rope BC is needed (which can be obtained from the start of the N2L and suvat approach).

To avoid this complication, it would be necessary to consider the motion up to the point where C hits the ground, and use an energy approach to find the speed of B at this point. Then, with the rope BC being slack, an energy approach can be used again to find the further distance moved by B.]

Once C hits the ground, the rope BC becomes slack, and B continues to move up the slope. Let d be the total distance moved by B.

Then total loss of GPE is

$15g(1.5) - 4gd - 5g(dsin40°) = 220.5 - 70.69659d$

And total work done against friction is

$\mu(5gcos40°)d + 15(1.5) = 15.01447d + 22.5$

The KE of A & B is zero at the start and end, but C has KE that has been dissipated:

KE of C when it hits the ground is

Original GPE of C less work done against friction, less work done against T_{BC} (from N2L & suvat approach)

From (3) in the N2L & suvat approach,

$15g - T_{BC} - 15 = 15a$, and $a = 1.92871$,

so that $T_{BC} = 103.06935$,

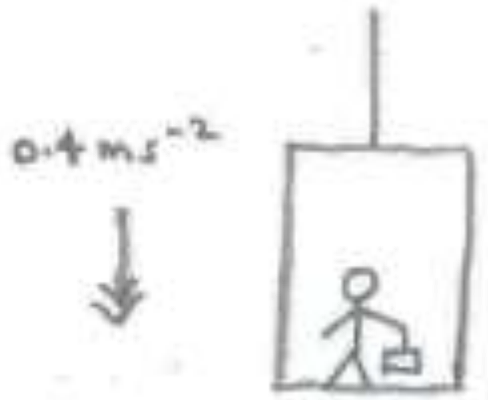
and KE of C when it hits the ground is

$15g(1.5) - 15(1.5) - 103.06935(1.5) = 43.39598\,J$

Then total loss of GPE equals total work done against friction plus KE dissipated,

so that $220.5 - 70.69659d = 15.01447d + 22.5 + 43.39598$, and $d = \frac{154.60402}{85.71106} = 1.80378m$

A man is in a lift, which is moving downwards with an acceleration of $0.4ms^{-1}$. The lift is suspended by a cable, and the man is holding a parcel by a light string, as in the diagram. The masses of the lift, man and parcel are 300kg, 80kg and 5kg, respectively.



(i) Find :

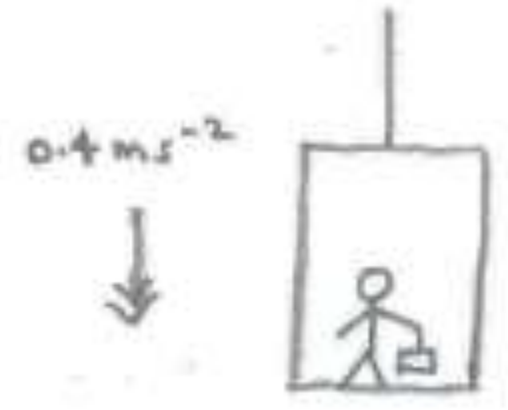
(a) the tension in the cable

(b) the reaction between the man and the floor of the lift

(c) the tension in the string

(ii) Does the man feel heavier or lighter than he would if the lift were stationary and he were no longer carrying the parcel?

A man is in a lift, which is moving downwards with an acceleration of $0.4ms^{-1}$. The lift is suspended by a cable, and the man is holding a parcel by a light string, as in the diagram. The masses of the lift, man and parcel are 300kg, 80kg and 5kg, respectively.



(i) Find :

(a) the tension in the cable

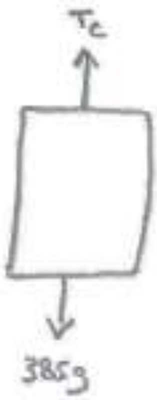
(b) the reaction between the man and the floor of the lift

(c) the tension in the string

(ii) Does the man feel heavier or lighter than he would if the lift were stationary and he were no longer carrying the parcel?

Solution

(a) Considering the lift, man and parcel as a single object:



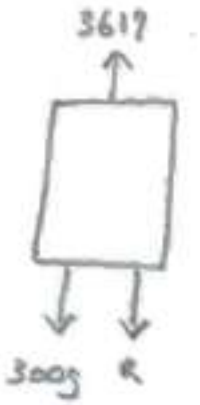
[T_C and 385g are the external forces]

$N2L \Rightarrow 385g - T_C = 385(0.4)$, where T_C is the tension in the cable

[any new symbols introduced need to be defined in an exam answer]

$\Rightarrow T_C = 385(9.8 - 0.4) = 3619\ N$

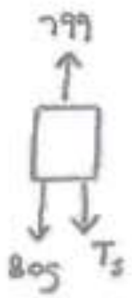
(b) Considering the forces on the lift:



$N2L \Rightarrow 300g + R - 3619 = 300(0.4)$, where R is the reaction between the man and the floor

$\Rightarrow R = 3619 + 120 - 300(9.8) = 799N$

(c) Considering the forces on the man:



$N2L \Rightarrow 80g + T_S - 799 = 80(0.4)$, where T_S is the tension in the string

$\Rightarrow T_S = 799 + 32 - 80(9.8) = 47N$

[Check: Considering the forces on the parcel:

$N2L \Rightarrow 5g - T_S = 5(0.4)$

$\Rightarrow T_S = 5(9.8) - 2 = 47N$]

(ii) If the lift is stationary and the man is not carrying the parcel, the reaction between himself and the floor is just his weight [see note below]: $80(9.8) = 784N$

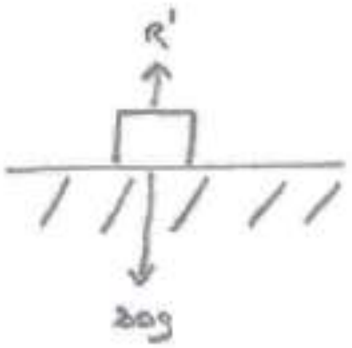
Thus he feels heavier, as $799 > 784$.

[The apparent gravity is now $9.8 - 0.4 = 9.4$, but the man's weight has effectively been increased by 5kg, giving a net apparent weight of

$85 \times 9.4 = 799N$ (this is a check on (b))]

Note: In the stationary situation (with no parcel),

$N2L \Rightarrow 80g - R' = 0 \Rightarrow R' = 80g$; ie the man's weight



A stepladder is made up of two sides, which have weights 80N and 8N. Both sides are of length 2m. There is a platform resting on the top, which together with a person standing on it weighs 700N. The two sides are also joined together by a horizontal light rope of length 1m, which starts at a distance of 0.6m along each side, from the base. See Fig. 1. There is no friction between the ladder and the ground, or between the platform and the ladder. Find the tension in the rope.

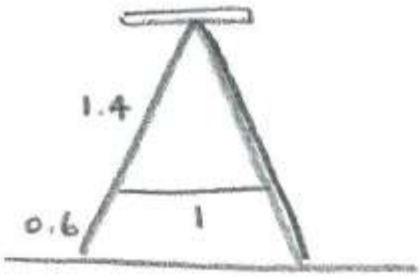


Fig. 1

A stepladder is made up of two sides, which have weights 80N and 8N. Both sides are of length 2m. There is a platform resting on the top, which together with a person standing on it weighs 700N. The two sides are also joined together by a horizontal light rope of length 1m, which starts at a distance of 0.6m along each side, from the base. See Fig. 1. There is no friction between the ladder and the ground, or between the platform and the ladder. Find the tension in the rope.

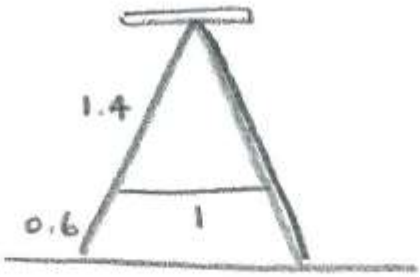


Fig. 1

Solution

[Equations can be obtained by applying N2L and/or taking moments for either individual components (eg one side of the stepladder), or the whole system. There will be a limit to the number of independent equations that can be created. But because some equations may prove to be redundant anyway (as we will see), it may not be worth attempting to ensure that all the equations are independent. Instead, we can just create any equations that look useful, and run the risk of some duplication. Not all of the equations created below are needed for the purpose of finding T, but may be of use for establishing other forces - or combinations of forces.]

Fig. 1a shows the various lengths involved.

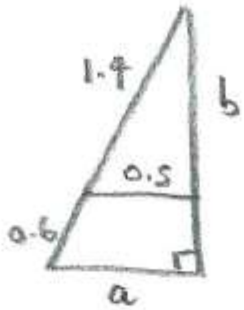


Fig. 1a

Considering the external forces on the ladder (including the rope, but excluding the platform) (Fig. 2),

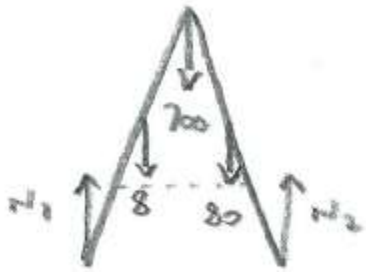


Fig. 2

$N_1 + N_2 = 788$ (1)

Also, taking moments about the base of the righthand side of the ladder :

$-N_1(2a) + (8)\left(\frac{3a}{2}\right) + 700a + 80\left(\frac{a}{2}\right) = 0$ (2),

so that $N_1 = \frac{1}{2}(12 + 700 + 40) = 376$, and hence $N_2 = 412$

Fig. 3 shows the force diagram for the lefthand side of the ladder (excluding the rope and the platform), where X & Y are the components of the reaction from the righthand side of the ladder, and R_1 is the reaction from the platform and person.

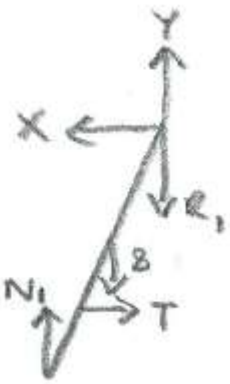


Fig.3

Horizontally this gives $T = X$ (3)

and vertically: $N_1 + Y = 8 + R_1$,

As $N_1 = 376,\ 368 + Y = R_1$ (4)

By N3L, the reaction forces on the righthand side of the ladder from the lefthand side are X & Y, as shown in Fig.4. And R_2 is the reaction from the platform and person.

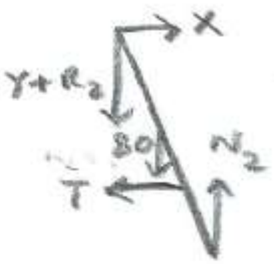


Fig. 4

Then, vertically: $N_2 = 80 + Y + R_2$

As $N_2 = 412,\ 332 = Y + R_2$ (5)

(Horizontally just reproduces $T = X$)

Fig. 5

Fig. 5 shows the forces on the platform and person, giving

$R_1 + R_2 = 700 \ \ (6)$

[Note that this could have been obtained by adding (4) & (5).]

In order to involve the location of T, we can take moments about the top of the ladder, for say the lefthand side, to give:

$-N_1 a + Tb + (8)\left(\frac{a}{2}\right) = 0 \ \ (6),$

so that $T = \frac{a}{b}(376 - 4) = \frac{372a}{b}$

Also, from Fig. 1a, by similar triangles, $\frac{a}{0.5} = \frac{2}{1.4}$, so that $a = \frac{5}{7}$,

and $b^2 = 1.4^2 - 0.5^2$, so that $b = \frac{\sqrt{171}}{10} = \frac{3\sqrt{19}}{10}$,

and hence $T = \frac{372(5)(10)}{7(3\sqrt{19})} = 203.197$

(i) A lift of mass $400kg$ has a maximum acceleration or deceleration of $1ms^{-2}$, and the lift cable can support a tension of $9000N$. What is the maximum number of people of mass $80kg$ that can safely be carried? (Assume $g = 10ms^{-2}$.)

(ii) If a single person of mass $80kg$ is in the lift when it is accelerating downwards at $1ms^{-2}$, how much lighter does the person feel, compared with their usual weight, as a percentage?

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(ii) If a single person of mass $80kg$ is in the lift when it is accelerating downwards at $1ms^{-2}$, how much lighter does the person feel, compared with their usual weight, as a percentage?

Solution

(i) When the lift is accelerating upwards (or decelerating downwards) at $1ms^{-2}$,

$N2L \Rightarrow T - (400 + 80n)g = (400 + 80n)(1)$

When the lift is accelerating downwards (or decelerating upwards) at $1ms^{-2}$,

$N2L \Rightarrow T - (400 + 80n)g = (400 + 80n)(-1)$

T is greater in the 1st case

When $T = 9000$, $9000 = 4400 + 880n$,

so that $n = 5.23$, and hence 5 people can safely be carried

(ii) Let R be the reaction of the lift floor on the person.

Then, considering the forces on the person,

$N2L \Rightarrow R - 80g = 80(-1)$

$\Rightarrow R = 720N$

Their usual weight is $80g = 800N$,

and so there has been a reduction of 10%.