Energy (5 pages; 26/1/19)

See also: Hooke's Law

(1) Types of Energy

<u>Mechanical</u>	
Kinetic	
Potential	Gravitational
	Elastic
<u>Non-mechanical</u>	Heat
	Sound
	Light
	Electrical
	Chemical
	Nuclear

A 'conservative' force is one for which mechanical energy is conserved; ie energy is swapped between kinetic and potential energy.

Friction is an example of a non-conservative (or 'dissipative') force.

(2) Conservation of Energy and the Work-Energy principle

The principle of Conservation of Energy states that $KE + PE =$ constant, provided that there are no external forces other than the forces giving rise to the PE (such as gravity or the tension in a spring etc.) Potential energy is the work expected to be done by a force, and any work done by gravity or the tension in the spring can be ignored, because it is already included in the change in PE (see Example 1 below).

The Work-Energy principle can be used when there are other external forces. It states that the total work done by all forces acting on an object = increase in the object's KE (see (iii) below). This total work should include any work done by gravity or the tension in a spring because this time the PE is not being included in the energy.

Sometimes there is a choice between using either the principle of Conservation of Energy or the Work-Energy principle, as shown by the following examples.

Example 1: Object sliding down a smooth slope; starting from rest and finishing at speed v. The object falls through a vertical height of h.

CoE: 0 (initial KE) + mgh (initial PE) = ½ mv^2 (final KE) + 0 (final PE)

WE: mg x h (work done by gravity) = ½ mv^2 (increase in KE)

Example 2: A cyclist rides downhill, doing 1000J of work against friction. His starting speed is u and his finishing speed is v. He moves through a vertical height of h.

CoE: can't be used, as there is an external force other than gravity (ie the friction)

WE: mg x h (work done by gravity) - 1000 (work done by friction, opposing motion) = ½ mv^2 - ½ mu^2 (increase in KE)

[Note that we considering the work done <u>on</u> the cyclist, so that the work done by the cyclist against friction does not feature directly in the equation.]

Example 3: An object is lifted through a distance of h; starting and finishing at rest.

CoE: can't be used, as there is an external force other than gravity

WE: mg x h (work done against gravity) + mg x (-h) (work done by gravity) = 0 (increase in KE)

(3) Derivation of the Work-Energy principle (showing why kinetic energy is in the form ½ mv^2)

(a) In the case of a constant force,

Work = Force x distance moved (in the direction of the force)

= ma . s

Then the suvat equation $v^2 = u^2 + 2as \Rightarrow$

Work = m . ½ (v^2 - u^2) = ½ mv^2 – ½ mu^2

(b) In the case of a variable force,

Work = $\int_a^b F(s)ds$, where a and b represent the start and end points (the integral is the limit of $\sum_a^b F(s) \, . \, \delta s$; ie force x small distance, summed; the force is written as F(s) to show that it is a function of the position)

Now, Newton's 2nd Law, F = ma can be written as F = m . $\frac{dv}{dt}$

and $\frac{dv}{dt} = \frac{dv}{ds} . \frac{ds}{dt} = \frac{dv}{ds}$. v , so that F = mv $\frac{dv}{ds}$

Hence, F(s) ds = mv . $\frac{dv}{ds}$ ds = mv dv

and Work = $\int_{v_a}^{v_b} mv \ dv$ (where v_a & v_b are the speeds at positions a & b respectively)

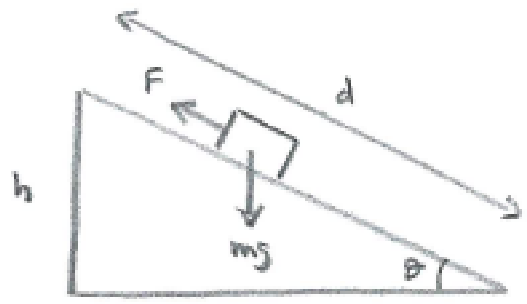
$= [\ ½\ \mathrm{m}\ v^2]_{v_a}^{v_b} = ½\ \mathrm{m}\ {v_b}^2 - ½\ \mathrm{m}\ {v_a}^2$

(4) Exercise

An object starts at rest and slides a distance d down a slope, against a frictional force F, as shown in the diagram. Find an expression for the final speed v:

(a) using the work-energy principle

(b) using conservation of energy, modified to take account of the friction



Solution

(a) By the work-energy principle, the gain in KE is equal to the work done by all the forces.

Work done by friction $= -Fd$ (as the force is in the opposite direction to the motion)

Work done by gravity $= (mgsin\theta)d$, where $mgsin\theta$ is the component of gravity in the direction of motion

$= mgh$

[**Note**: This can also be thought of as: the force (mg) × the distance moved in the direction of the force (h)]

Then $\frac{1}{2}mv^2 - 0 = mgh - Fd$

and so $v = \sqrt{2gh - \frac{2Fd}{m}}$

(b) Modifying the principle of conservation of energy,

initial KE of object + initial PE of object + work done on object

= final KE of object + final PE of object

So $0 + mgh - Fd = \frac{1}{2}mv^2 + 0$,

so that $\frac{1}{2}mv^2 = mgh - Fd$, as before

Note: the $-Fd$ can alternatively be thought of as the energy lost by the object in doing work against friction.