

## Ellipses - Exercises (1 page; 17/2/20)

See also the separate note "Ellipses" for further exercises.

### Key to difficulty:

\* easier

\*\* moderate

\*\*\* harder

(1\*\*) Show that the equation of the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ at the point } (x_1, y_1) \text{ is } \frac{yy_1}{b^2} + \frac{xx_1}{a^2} = 1$$

(2\*\*\*) Given the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and circle  $x^2 + y^2 = a^2$ , let  $l_1$  be the tangent to the ellipse at the point  $(a\cos\theta, b\sin\theta)$  and  $l_2$  be the tangent to the circle at the point  $(a\cos\theta, a\sin\theta)$ . Find the locus of the point of intersection of  $l_1$  &  $l_2$ , as  $\theta$  varies.

(3\*\*\*) Show that the area within the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $\pi ab$