Discrete Random Variables Q3 [Practice/E](9/6/21)

(i) Show algebraically that $E[aX + b] = aE(X) + b$

(ii) Show that $Var(X) = E(X^2) - \mu^2$

(iii) Show that $Var(aX + b) = a^2 VarX$

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Solution

(i) $E[aX + b] = \sum_x (ax + b)P(X = x)$

$= [a \sum_x xP(X = x)] + b \sum_x P(X = x)$

$= aE(X) + b$

(ii) $Var(X) = E[(X - \mu)^2]$

$= E[X^2 - 2X\mu + \mu^2]$

$= \sum_x (x^2 - 2x\mu + \mu^2)P(X = x)$

$= [\sum_x x^2 P(X = x)] - [2\mu \sum_x xP(X = x)] + \mu^2 \sum_x P(X = x)$

$= E(X^2) - 2\mu E(X) + \mu^2$

$= E(X^2) - 2\mu^2 + \mu^2$

$= E(X^2) - \mu^2$

(iii) $Var(aX + b) = E[(aX + b)^2] - [E(aX + b)]^2$

$= E[a^2X^2 + 2abX + b^2] - [aE(X) + b]^2$

$= a^2E(X^2) + 2abE(X) + b^2 - [a^2[E(X)]^2 + 2abE(X) + b^2]$

$= a^2E(X^2) - a^2[E(X)]^2$

$= a^2 VarX$