Discrete Random Variables Q1 [8 marks] (9/6/21)

Exam Boards

OCR : Statistics (Year 1)

MEI: Statistics a

AQA: Statistics (Year 1)

Edx: S1 (Year 1)

The probability distribution of a discrete random variable is shown below.

x	1	2	3
$P(X = x)$	p	q	r

Given that $E(X) = \frac{7}{3}$ and $Var(X) = \frac{5}{9}$, find p, q and r. [8 marks]

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Solution

$p + q + r = 1$ (1) [1 mark]

$E(X) = \frac{7}{3} \Rightarrow 1(p) + 2(q) + 3(r) = \frac{7}{3}; 3p + 6q + 9r = 7$ (2)

[2 marks]

$Var(X) = \frac{5}{9} \Rightarrow E(X^2) - [E(X)]^2 = \frac{5}{9}$

$\Rightarrow 1(p) + 4(q) + 9(r) - \frac{49}{9} = \frac{5}{9}$

$\Rightarrow p + 4q + 9r = 6$ (3) [2 marks]

Using (1) to eliminate r from (2) & (3):

$3p + 6q + 9(1 - \mathrm{p} - \mathrm{q}) = 7;\ -6p - 3q = -2; 6p + 3q = 2$ (4)

and $p + 4q + 9(1 - \mathrm{p} - \mathrm{q}) = 6;\ -8p - 5q = -3;$

$8p + 5q = 3$ (5)

Then $4(4) - 3(5) \Rightarrow -3q = -1; q = \frac{1}{3}$

and $(4) \Rightarrow 6p = 2 - 1; p = \frac{1}{6}$

and $(1) \Rightarrow r = 1 - \frac{1}{6} - \frac{1}{3} = \frac{1}{2}$ [3 marks]