Discrete Random Variables - Exercises (2 pages; 14/2/26)

(1) The probability distribution of a discrete random variable is shown below.

x	1	2	3
$P(X = x)$	p	q	r

Given that $E(X) = \frac{7}{3}$ and $Var(X) = \frac{5}{9}$, find p, q and r.

(2) The random variables X_i , for $i = 1$ to 100, are independent, and $P(X_i = 1) = P(X_i = -1) = \frac{1}{2}$

Find:

(i) $Var(X_1)$

(ii) $Var(X_1 + X_2 + \cdots + X_{100})$

(iii) $Var(100X_1)$

(iv) $Var(X_1 - X_2)$

(3)(i) Show algebraically that $E[aX + b] = aE(X) + b$

(ii) Show that $Var(X) = E(X^2) - \mu^2$

(iii) Show that $Var(aX + b) = a^2 VarX$

(4) Show that $\sum_{x=0}^{n} P(X = x) = 1$ for the Binomial distribution.

(5) If $X \sim B(n, p)$, prove that $E(X) = np$