

## Dijkstra's Algorithm Exercise – with Sol'n

(2 pages; 12/3/26)

(i) Given that, for Dijkstra's algorithm, the work involved at each step is approximately proportional to the number of vertices that have not been permanently labelled, show that Dijkstra's algorithm has quadratic order.

(ii) A table of shortest distances between all 51 cities in England has been constructed by applying Dijkstra's algorithm multiple times. The program that created the table took 120 seconds to run. The table is now to be extended to include the other 18 cities in the UK. How long will the program take to produce the new table?

## Solution

$$\begin{aligned} \text{(i) Total work} &\approx (n - 1) + (n - 2) + (n - 3) + \cdots + 1 \\ &= \frac{1}{2}(n - 1)n \end{aligned}$$

As this is a quadratic function of  $n$ , the algorithm has quadratic order.

(ii) Applying Dijkstra's algorithm  $n - 1$  times [condone  $n$ ] (for  $n$  cities)  $\Rightarrow$  cubic complexity

$$\Rightarrow 120 \times \left(\frac{69}{51}\right)^3 = 297 \text{ seconds, for UK}$$

but only approximate, so 300 seconds