Differentiation – Exercises (9 pages; 31/1/26)

Find the derivative of $tanx$ using (a) the Quotient rule, and (b) the Product rule.

Solution

(a) $\frac{d}{dx}(tanx) = \frac{d}{dx}\left(\frac{sinx}{cosx}\right) = \frac{cosx(cosx) - sinx(-sinx)}{cos^2x}$

$= (cos^2x + sin^2x)sec^2x$

$= sec^2x$

(b) $\frac{d}{dx}(tanx) = \frac{d}{dx}(sinx\,.\,(cosx)^{-1})$

$= cosx(cosx)^{-1} + (sinx)(-1)(cosx)^{-2}(-sinx)$

$= 1 + tan^2x = sec^2x$

Show that $\int \frac{1}{\sqrt{1+a^2x^2}} dx = \frac{1}{a} ln\left|\sqrt{1+a^2x^2} + ax\right| + c$, by differentiation.

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Solution

$$\frac{d}{dx}\left(\frac{1}{a}ln\left|\sqrt{1+a^2x^2} + ax\right|\right) = \frac{1}{a} \cdot \frac{\frac{1}{2}(1+a^2x^2)^{-\frac{1}{2}}(2a^2x)+a}{\sqrt{1+a^2x^2}+ax}$$

Writing $A = \sqrt{1+a^2x^2}$, this gives:

$\frac{A^{-1}ax+1}{A+ax} = \frac{1}{A} \cdot \frac{ax+A}{A+ax} = \frac{1}{A}$, as required

If $f(x) = x^2$, what is $f'(3x)$?

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Solution

Method 1

Note that the differentiation is wrt $3x$ (rather than x).

Let $u = 3x$. Then $f'(3x) = f'(u) = \frac{d}{du}(u^2) = 2u = 2(3x) = 6x$

Method 2

$f'(x) = 2x \Rightarrow f'(3x) = 2(3x) = 6x$

Find the turning points of $y = (x^2 - 4x + 3)^2$

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Solution

Method 1

As $x^2 - 4x + 3 = (x-1)(x-3)$,

$y = (x-1)^2(x-3)^2$

Then $\frac{dy}{dx} = 2(x-1)(x-3)^2 + (x-1)^2(2)(x-3)$

$= 2(x-1)(x-3)(x-3+x-1)$

$= 4(x-1)(x-3)(x-2)$

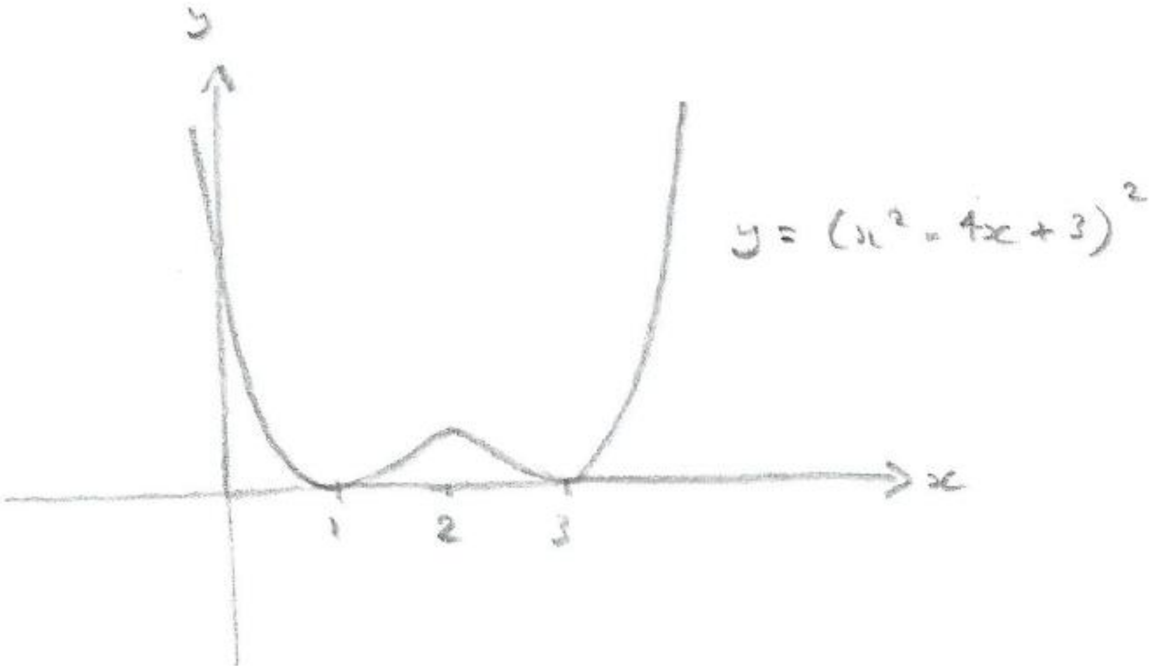
$\frac{dy}{dx} = 0$ when $x = 1, 2 \,\&\, 3$

At $x = 1$, $\frac{dy}{dx}$ changes from –ve to +ve, indicating a min. point.

At $x = 2$, $\frac{dy}{dx}$ changes from +ve to -ve, indicating a max. point.

At $x = 3$, $\frac{dy}{dx}$ changes from –ve to +ve, indicating a min. point.

The min. points are therefore (1, 0) and (3, 0), whilst the max. is at (2,1).



Method 2

$(x^2 - 4x + 3)^2 \geq 0$ and $(x^2 - 4x + 3)^2 = (x - 1)^2(x - 3)^2 = 0$ has roots at $x = 1$ & 3, so that there are minima at these two points.

For $x = 1 - t$, $y = x^2 - 4x + 3$ and hence y = $(x^2 - 4x + 3)^2$ increases as t increases, and similarly for $x = 3 + t$.

For $1 < x < 3$, y = $(x^2 - 4x + 3)^2$ attains a max. when

$x^2 - 4x + 3$ (which is negative in this range) is at a min. ; ie when

$$x = 2.$$