Numerical methods: Differential Equations

Exercises (1 page; 8/3/26)

(1) The function $y(x)$ satisfies the differential equation

$\frac{d^2y}{dx^2} = \frac{ye^x}{x+1}$, subject to the initial conditions $y = 2$ and $\frac{dy}{dx} = 1$ when $x = 0$.

Use 2 iterations of the approximations $\left(\frac{d^2y}{dx^2}\right)_{n+1} \approx \frac{y_n + y_{n+2} - 2y_{n+1}}{h^2}$

and $\left(\frac{dy}{dx}\right)_{n+1} \approx \frac{y_{n+2} - y_n}{2h}$ to estimate the value of y when $x = 0.4$

(2) The function $y(x)$ satisfies the differential equation

$\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} = x^2 + 2y,$

subject to the boundary conditions $y = 2$ and $\frac{dy}{dx} = 4$ when $x = 1$.

Use 2 iterations of the approximations $\left(\frac{d^2y}{dx^2}\right)_{n+1} \approx \frac{y_n + y_{n+2} - 2y_{n+1}}{h^2}$

and $\left(\frac{dy}{dx}\right)_{n+1} \approx \frac{y_{n+2} - y_n}{2h}$ to estimate the value of y when $x = 1.2$