Convexity and concavity (3 pages; 29/8/18)

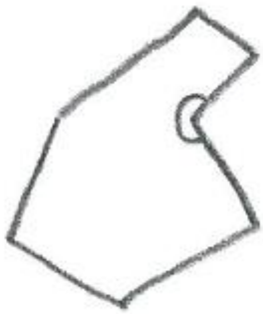
(1) A convex polygon is the usual type (such as a regular pentagon), where the sides all bulge outwards, and all of the interior angles are less than 180°. In optics, a convex lens also bulges outwards.

With a concave polygon, there is at least one interior angle greater than 180°. A concave lens bulges inwards.

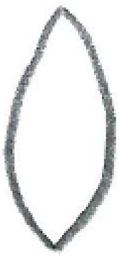
In this way, we can talk about convex or concave regions.

A convex **region** can be defined as one where the line connecting any two points in the region is itself within the region.

convex polygon



concave polygon



convex lens

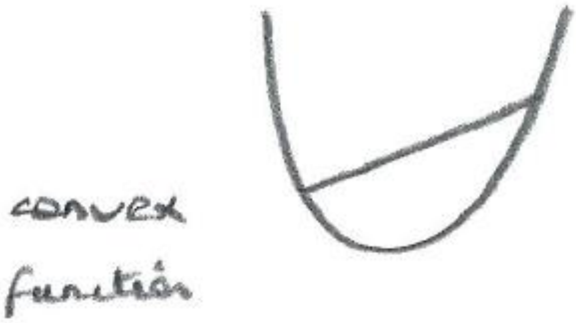
concave lens

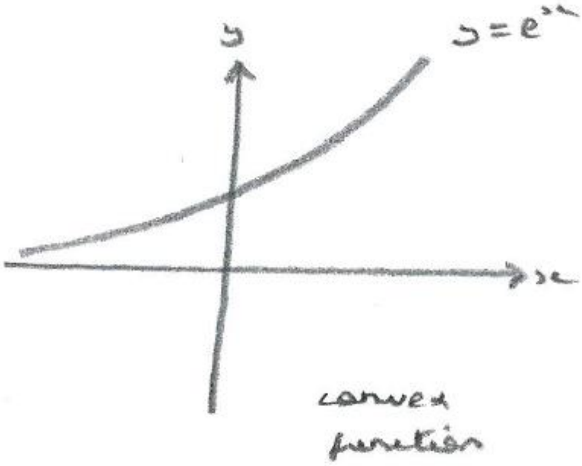
(2) An example of a convex **function** is $y = x^2$. The use of the term 'convex' seems to be fairly arbitrary, but relates to the fact that the region **above** the curve is convex. Thus, the line connecting any two points on the curve lies entirely above the curve. Expressing this mathematically, a function is convex when

$f(\lambda a + (1-\lambda)b) \leq \lambda f(a) + (1-\lambda)f(b)$ for any a & b in the domain of the function $y = f(x)$; ie the curve lies below the line joining a & b

Also, a function is convex when $\frac{d^2y}{dx^2} \geq 0$ (or $\frac{d^2y}{dx^2} > 0$ for a strictly convex function).

Another example of a convex function is $y = e^x$ (think of conve^x).



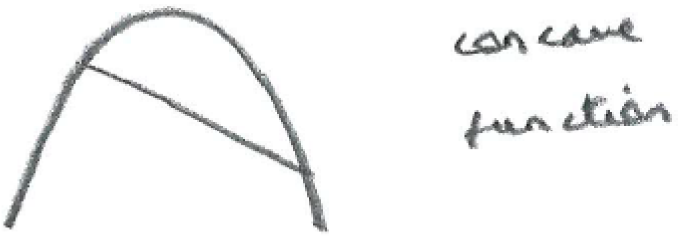


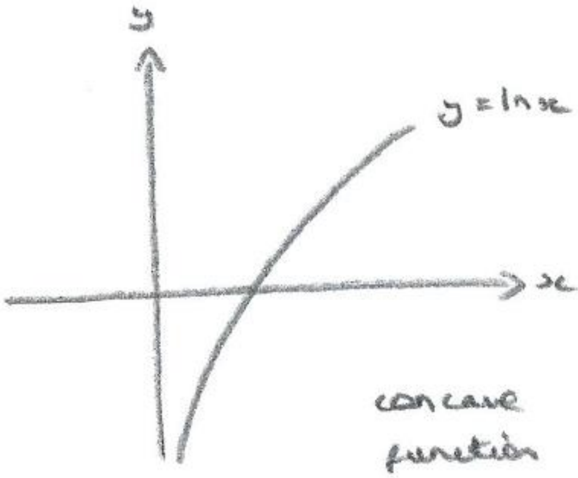
(3) Examples of concave functions are $y = -x^2$ and $y = lnx$.

Mathematically, a function is concave when $\frac{d^2y}{dx^2} \leq 0$.

Alternatively, a function is concave when

$f(\lambda a + (1-\lambda)b) \geq \lambda f(a) + (1-\lambda)f(b)$ for any a & b in the domain of the function $y = f(x)$; ie the curve lies above the line joining a & b





(4) A point of inflexion occurs when $f''(x)$ changes sign. This is when a function changes from convex to concave (or vice-versa).

[See separate note "Turning points and points of inflexion".]

(5) Note: The MEI A Level exams (and possibly others) use terminology which is hard to justify, and tends to contradict the conventional uses of the expressions 'concave' and 'convex', described later on. The following is from the MEI specification: