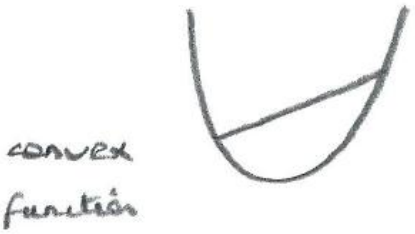
Convex & Concave Functions (3 pages; 3/11/26)

(1) For a convex function, the line connecting any two points on the curve lies entirely above the curve.

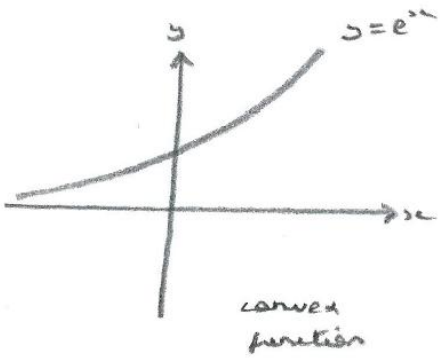


So when using the Trapezium method for approximating the area under a curve, there will be an overestimate for any convex parts of the curve.

(2) Examples of convex functions are:

$y = x^2$

$y = e^x$ (think of conve^x) and $y = e^{-x}$



(3) The definition of convexity can be expressed mathematically as: $f(\lambda a + (1-\lambda)b) < \lambda f(a) + (1-\lambda)f(b)$

for any a & b in the domain of the function $y = f(x)$; ie the curve lies below the line joining a & b

(4) For a concave function, the line connecting any two points on the curve lies entirely below the curve.

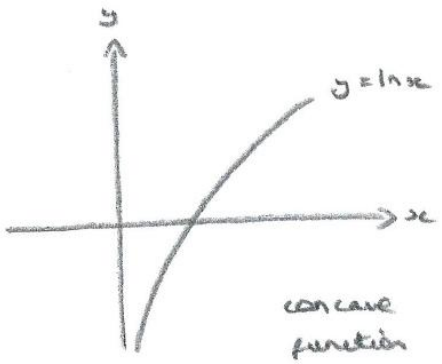


(5) Examples of concave functions are:

$y = -x^2$

$y = lnx$ (for $x > 0$)

and (its reflection in the y-axis) $y = \ln(-x)$ (for $x < 0$)



(6) The definition of concavity can be expressed mathematically as $f(\lambda a + (1-\lambda)b) > \lambda f(a) + (1-\lambda)f(b)$

for any a & b in the domain of the function $y = f(x)$; ie the curve lies above the line joining a & b

(7) Alternatively, a function is convex when $\frac{d^2y}{dx^2} > 0$, and concave when $\frac{d^2y}{dx^2} < 0$.

A point of inflexion occurs when $\frac{d^2y}{dx^2}$ changes sign. This is when a function changes from convex to concave (or the reverse).