Conditional Probability (2 pages; 1/2/23)

(1) By considering regions in a Venn diagram, $P(B|A) = \frac{P(A\cap B)}{P(A)}$.

Alternatively, $P(A \cap B) = P(A)P(B|A)$.

(2) A and B may or may not occur simultaneously. Examples are:

(a) A = pupil is late to school on Monday; B = pupil is late to school on Tuesday

(b) A = it is raining in London; B = it is raining in New York

The formula $P(A \cap B) = P(A)P(B|A)$ is easier to understand if A occurs before B, but is also valid if A and B take place at the same time. In the case of (b), for example, we could first of all consider the weather in London, and then the weather in New York (given that it is raining in London).

(3) Events A and B are independent when A doesn't influence B (and vice-versa), so that $P(B|A) = P(B)$ (1)

Then $P(B) = P(B|A) = \frac{P(A\cap B)}{P(A)}$,

so that $P(A \cap B) = P(A)P(B)$ (2)

Either (1) or (2) can be used as a test for independence.

(4) Sometimes independence isn't immediately obvious. For example, suppose that the following 2-way table applies in connection with pupils choice of subjects:

	Italian	German	Spanish	Total
Male	18	14	8	40
Female	23	21	16	60
Total	41	35	24	100

Then $P(M) = \frac{40}{100} = \frac{2}{5}$ and $P(M|S) = \frac{8}{24} = \frac{1}{3}$

so that $P(M|S) \neq P(M)$ and hence M and S aren't independent.

But $P(M|G) = \frac{14}{35} = \frac{2}{5}$, so that M and G are independent.

Alternatively, $P(S) = \frac{24}{100} = \frac{6}{25}$, $P(M \cap S) = \frac{8}{100} = \frac{2}{25}$, and

$P(M).P(S) = \frac{2}{5}.\frac{6}{25} = \frac{12}{125}$, and so M and S aren't independent, as

$P(M \cap S) \neq P(M).P(S)$

Also, $P(G) = \frac{35}{100} = \frac{7}{20}$, $P(M \cap G) = \frac{14}{100} = \frac{7}{50}$, and

$P(M).P(G) = \frac{2}{5}.\frac{7}{20} = \frac{7}{50}$, and so M and G are independent, as

$P(M \cap G) = P(M).P(G)$

(5) Mutually exclusive events

This shouldn't be confused with independence.

In the above example, G and S are mutually exclusive, as
$P(G \cap S) = 0$

But G and S are not independent, as $P(G|S) = 0 \neq P(G)$.

Alternatively, $P(G \cap S) = 0 \neq P(G).P(S)$