

## Complex Numbers Overview (22/6/21)

### Q1 [Practice/E]

Represent the following on the Argand diagram:

(i)  $|z - i| > |z + 1|$  [5 marks]

(ii)  $|z - i| = 2|z + 1|$  [6 marks]

### Q2 [6 marks]

$1 + 3i$  is a root of the equation  $z^3 + pz + q = 0$  (where  $p$  &  $q$  are real). Find the other roots, and the values of  $p$  &  $q$ .

### Q3 [5 marks]

Find the square roots of  $3 - 4i$

### Q4 [4 marks]

Find the solutions of  $z^2 = i$

### Q5 [7 marks]

Find  $(1 + i)^{10}$

### Q6 [4 marks]

Find the equation of the line satisfying

$$|z + 10| = |z - 6 - 4i\sqrt{2}|$$

**Q7** [5 marks]

Express  $(1 - i)^6$  in the form  $x + iy$

**Q8** [8 marks]

Find the cube roots of  $-8$  in cartesian form

**Q9** [Practice/Y2/H]

Consider two roots of  $z^n = \cos\theta + i\sin\theta$  :

$$z_r = \cos\left(\frac{\theta}{n} + \frac{2\pi r}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi r}{n}\right)$$

$$\text{and } z_R = \cos\left(\frac{\theta}{n} + \frac{2\pi R}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi R}{n}\right)$$

(i) Find the condition on  $n$  for  $z_R$  to equal  $-z_r$  for some  $R$ , and find  $R$  in terms of  $r$ .

(ii) Find the condition on  $\theta$  for  $z_R$  to be the conjugate of  $z_r$  for some  $R$ , and find  $R$  in terms of  $r$ .

**Q10** [7 marks]

Let  $z = \frac{a+i}{1+ai}$ . If  $\arg z = -\frac{\pi}{4}$ , find the possible values of  $a$

**Q11** [8 marks]

Find the modulus and argument of  $e^{\frac{7\pi i}{10}} - e^{\frac{9\pi i}{10}}$

**Q12** [Practice/Y1/E]

For each of the following numbers, say whether they are imaginary or complex (or both):

- (i) 1 (ii)  $i$  (iii) 0 (iv)  $1 + i$

**Q13** [Practice/Y1/E]

Are these statements true or false? (Give an explanation, or a counter example, as appropriate.)

- (i) All imaginary numbers are complex numbers.
- (ii) All complex numbers are imaginary numbers.
- (iii) All real numbers are complex numbers.
- (iv) Zero is an imaginary number.
- (v) The imaginary part of a complex number is an imaginary number.
- (vi) All complex numbers are either real numbers or imaginary numbers.
- (vii) Two imaginary numbers added together can sometimes give a real number.
- (viii) If two complex numbers multiply to give a real number, then they must be conjugates of each other.
- (ix) The square root of a non-real complex number is never real.

**Q14** [Practice/Y1/E]

- (i) How are the complex numbers  $z$  and  $zi$  related to each other geometrically?

(ii) How are the complex numbers  $z$  and  $\frac{1}{z}$  related to each other geometrically?

**Q15** [Practice/Y1/E]

Find  $(2 + 5i) \div (1 + 3i)$  by (a) equating real and imaginary parts, and (b) another method

**Q16** [Practice/Y1/E]

Solve the equation  $(2 + i)z + 3 = 0$  by (a) equating real and imaginary parts, and (b) another method

**Q17** [Practice/Y1/E]

Solve the equation  $z^2 - 2z + 2 = 0$

- (a) by completing the square
- (b) by equating real & imaginary parts

**Q18** [Practice/Y1/M]

Find  $\arg \left\{ -\sin \left( \frac{\pi}{3} \right) + i \cos \left( \frac{\pi}{3} \right) \right\}$

**Q19** [Practice/Y1/M]

How are the complex numbers  $\cos\theta + i\sin\theta$  and  $\sin\theta + i\cos\theta$  related?