Circular Motion Exercises (Solutions) (2 pages; 21/1/20)

(1***) Find the height above the earth's surface of a satellite in geostationary orbit (above the equator), using the following data:

radius of earth = 6370 km

mass of earth $\approx$ 6 x 10^{24}kg

$G \approx$ 7 x 10^{-11}

Gravitational force $= \frac{GMm}{r^2}$

Solution

Steps:

(i) Set up F = ma

(ii) Establish ω

(iii) Solve for r

$\frac{GMm}{r^2} = m\omega^2 r$,

where M is the mass of the Earth, m is the mass of the satellite, r is the distance of the satellite from the Earth's centre, and ω is its angular speed

$\omega = \frac{2\pi}{24\times 3600}\ \ rads^{-1}$

Hence $r^3 = \frac{GM}{\omega^2} \approx \frac{6.67\times 10^{-11}\times 5.98\times 10^{24}\times(24\times 3600)^2}{(2\pi)^2} = 7.5421 \times 10^{22}$

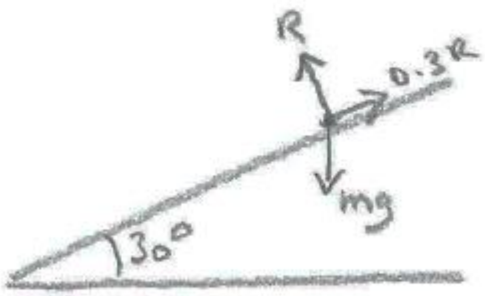
and $r = 4.22504 \times 10^7\ m$; ie 42250 km

Thus, height above Earth's surface is

$42250 - 6380 = 35900\ km\ (2sf)$

(2***) A bike is being ridden round a circular track of radius 50m, banked at 30°. If the coefficient of friction is 0.3, what is the slowest speed possible?

Solution



Vertical equilibrium ⇒

$Rcos30° + 0.3Rsin30° = mg\ \ (1)$

Circular motion ⇒ $Rsin30° - 0.3Rcos30° = \frac{mv^2}{50}\ \ (2)$

$(1)\&(2) \Rightarrow R = \frac{mg}{cos30° + 0.3sin30°} = \frac{mv^2}{50(sin30° - 0.3cos30°)}$

$\Rightarrow v^2 = \frac{9.8(50)(0.5 - 0.3\left(\frac{\sqrt{3}}{2}\right))}{\frac{\sqrt{3}}{2} + 0.3(0.5)} = \frac{117.694}{1.01603} = 115.837$

$\Rightarrow v = 10.762 = 10.8\ ms^{-1}$ (3sf)