Centre of Mass – Q4 [4 marks](1/6/21)
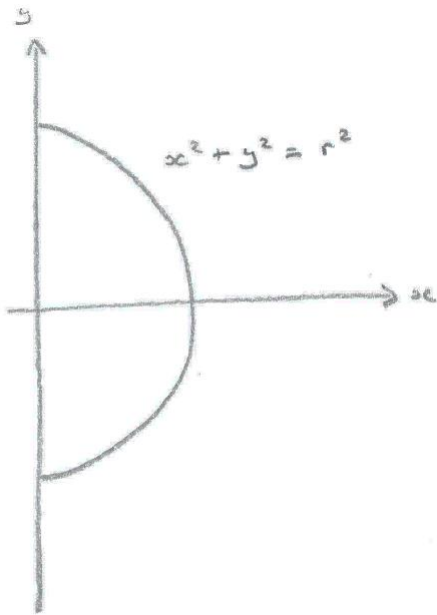
Exam Boards

OCR : Mechanics (Year 2)

MEI: Mechanics b
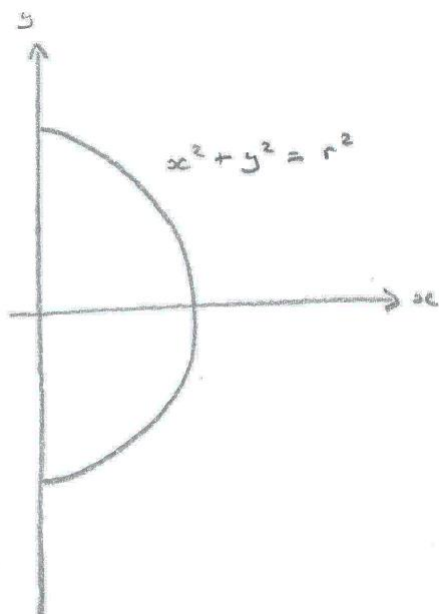
AQA: Mechanics (Year 2)

Edx: Mechanics 2 (Year 2)

Find the centre of mass of the semi-circular lamina shown in the diagram. [4 marks]

y

$x^2 + y^2 = r^2$

x

Find the centre of mass of the semi-circular lamina shown in the diagram. [4 marks]

Solution

By symmetry, we need only consider the top half [1 mark]

$\bar{x} = \frac{1}{\frac{1}{4}\pi r^2} \int_0^r xy \, dx = \frac{4}{\pi r^2} \int_0^r x\sqrt{r^2 - x^2} \, dx$ [1 mark]

$= \frac{-2}{\pi r^2} \int_0^r (-2x)\sqrt{r^2 - x^2} \, dx$

$= \frac{-2}{\pi r^2} \left[\frac{(r^2-x^2)^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^r = -\frac{4}{3\pi r^2}(0 - r^3) = \frac{4r}{3\pi}$ [2 marks]