Centre of Mass – Q2 [8 marks] (1/6/21)

Exam Boards

OCR : Mechanics (Year 2)

MEI: Mechanics b

AQA: Mechanics (Year 2)

Edx: -

The region between the curve $y = x^3 - x^2$ and the x-axis is rotated by 360° about the x-axis. Find the centre of mass of the solid of revolution obtained. [8 marks]

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Solution

By symmetry, $\bar{y} = 0$ [1 mark]

$y = x^3 - x^2 = x^2(x-1)$ meets the x-axis at $x = 0$ & $x = 1$

[1 mark]

$V\bar{x} = \int_0^1 x(\pi y^2 dx)$, where $V = \int_0^1 \pi y^2 dx$ [1 mark]

[$\pi y^2 dx$ is the volume of the disc of width dx at position x]

Thus $V = \pi \int_0^1 x^4(x-1)^2 dx = \pi \int_0^1 x^6 - 2x^5 + x^4 \, dx$ [1 mark]

$= \pi[\frac{1}{7}x^7 - \frac{2}{6}x^6 + \frac{1}{5}x^5]_0^1$

$= \pi\left(\frac{1}{7} - \frac{2}{6} + \frac{1}{5}\right) = \frac{(30-70+42)\pi}{210} = \frac{\pi}{105}\ units^3$ [2 marks]

And $\bar{x} = 105 \int_0^1 x^7 - 2x^6 + x^5 \, dx$

$= 105[\frac{1}{8}x^8 - \frac{2}{7}x^7 + \frac{1}{6}x^6]_0^1$

$= 105\left(\frac{1}{8} - \frac{2}{7} + \frac{1}{6}\right) = \frac{105(21-48+28)}{168}$

$= \frac{105}{168} = \frac{35}{56} = \frac{5}{8} = 0.625$ [2 marks]

Thus the centre of mass is $(0.625, 0)$