

Critical Path Analysis - Exercises (with Sol'ns)

(14 pages; 20/2/26)

(1)(i) Draw an activity-on-arc (MEI/OCR/Edx) or activity-on-node (AQA) network to represent the following project, and establish the earliest and latest event times.

Activity	Immediate predecessors	Duration
A	-	5
B	-	7
C	B	4
D	B	6
E	D	5
F	A	3
G	F	9
H	A,C	7
I	E	7
J	F	5
K	D,G,H,I	8
L	D,G,H,I	8
M	E	9
N	J	6
O	L,M	9
P	O	6

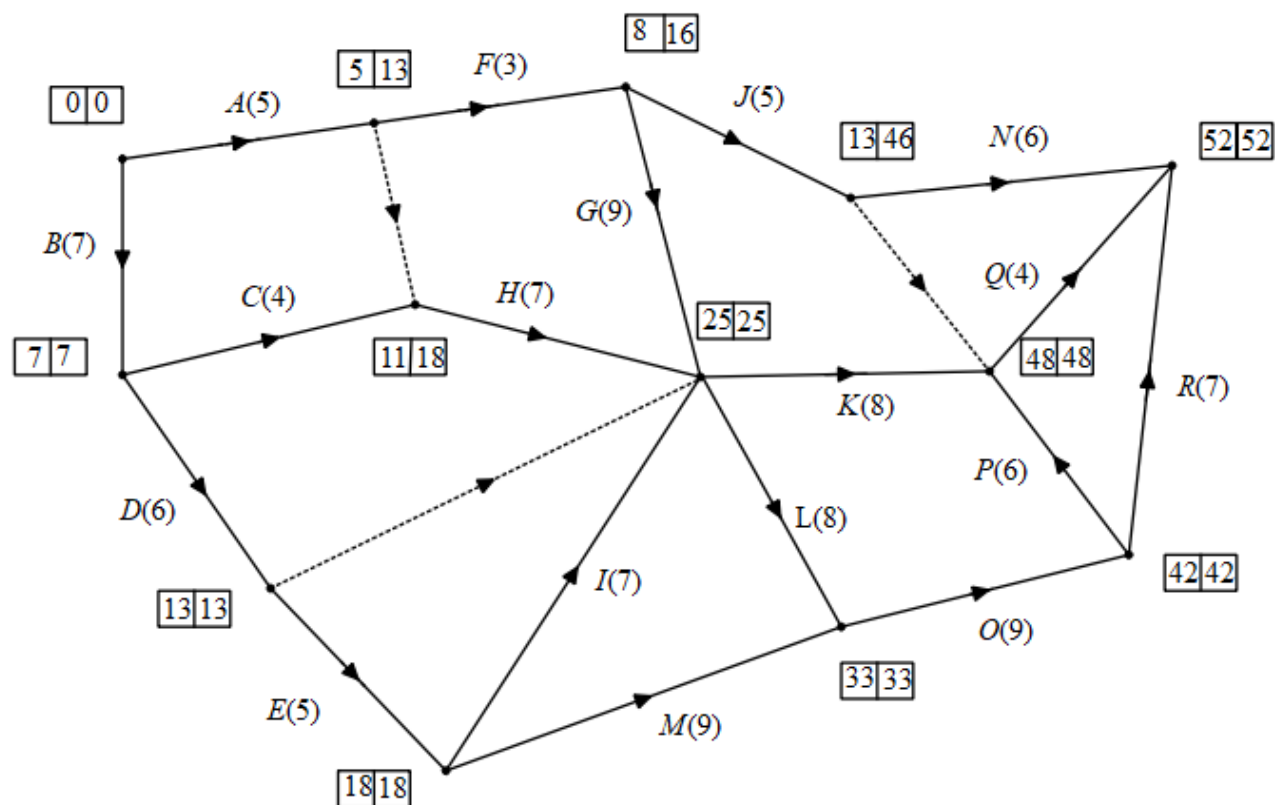
Q	J,K,P	4
R	O	7

(ii) State the critical path(s).

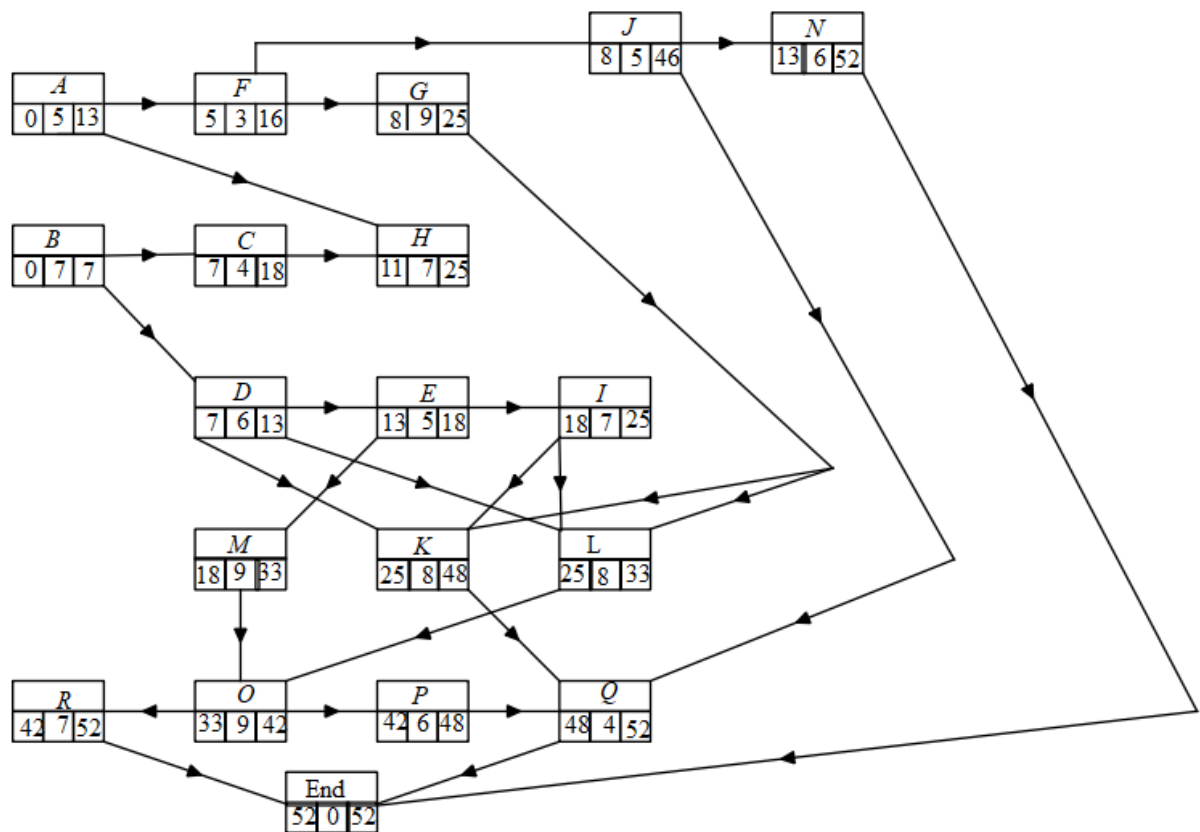
(iii) The duration of each activity can be reduced at a cost (with the amount of possible reduction varying by activity). If the project is to be completed as quickly as possible, describe how the required extra cost can be established.

Solution

(i) (MEI/OCR/Edx)



(i) (AQA)



(ii) Minimum completion time is 52.

Critical path is BDEILOPQ

(iii) Create the network based on the minimum durations, and then increase the durations as far as possible for the non-critical activities.

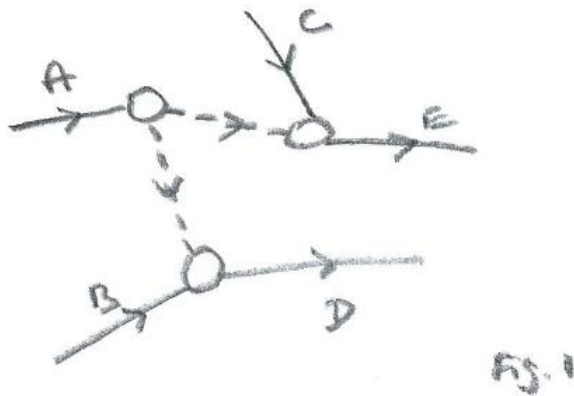
(2) Construct an activity network for the following part of a precedence table:

	depends on
A	-
B	-
C	-
D	A, B
E	A, C

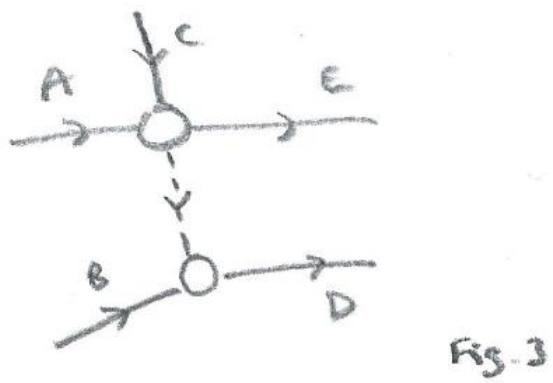
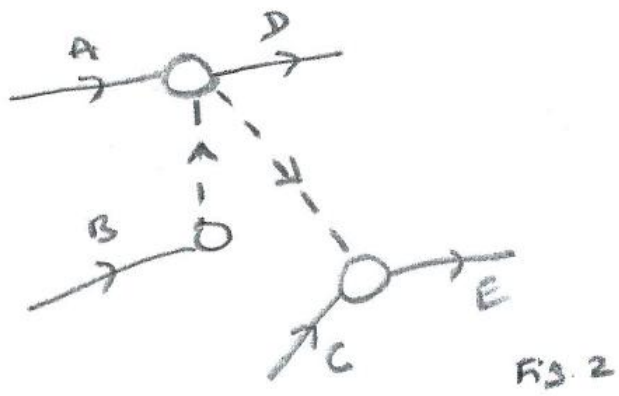
(2) Construct an activity network for the following part of a precedence table:

	depends on
A	—
B	—
C	—
D	A, B
E	A, C

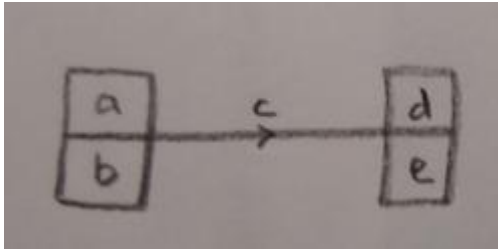
Solution



Note: Fig. 2 (below) doesn't work, because it implies that E depends on B. Also, Fig. 3 doesn't work, because it implies that D depends on C.



(3)

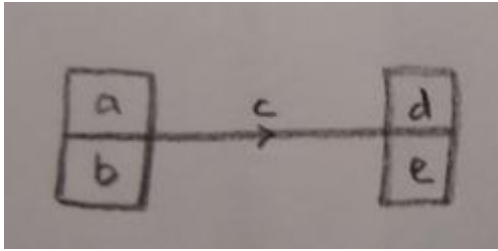


The diagram shows the earliest and latest event times relating to a particular activity, together with its duration.

(i) Create an example of a network to show that it is not necessary that $d \geq b$.

(ii) List the constraints that do apply.

(3)



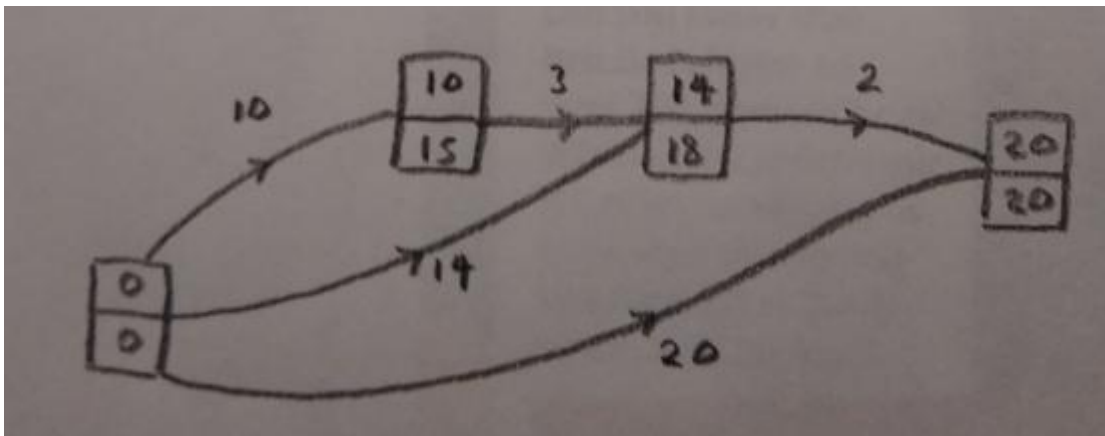
The diagram shows the earliest and latest event times relating to a particular activity, together with its duration.

(i) Create an example of a network to show that it is not necessary that $d \geq b$.

(ii) List the constraints that do apply.

Solution

(i)

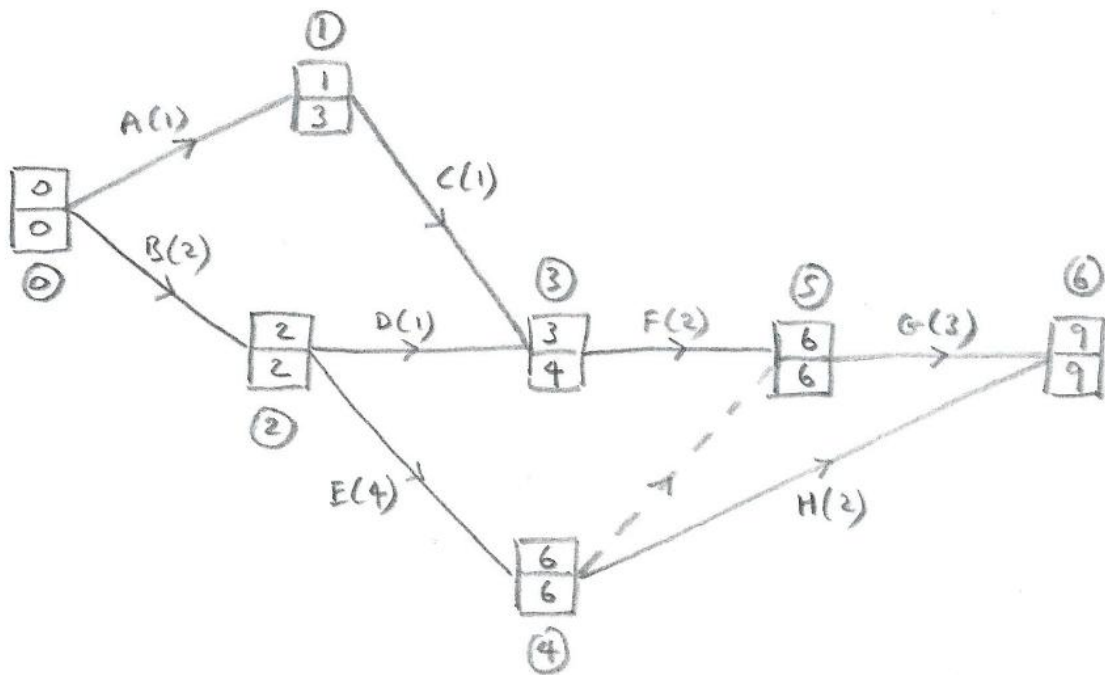


(ii) $a \leq b, d \leq e, d \geq a + c, e \geq b + c$

(4) Create an activity-on-arc network (with earliest and latest event times) for the following precedence table:

Activity	Duration	Immediately preceding activities
A	1	-
B	2	-
C	1	A
D	1	B
E	4	B
F	2	C,D
G	3	E,F
H	2	E

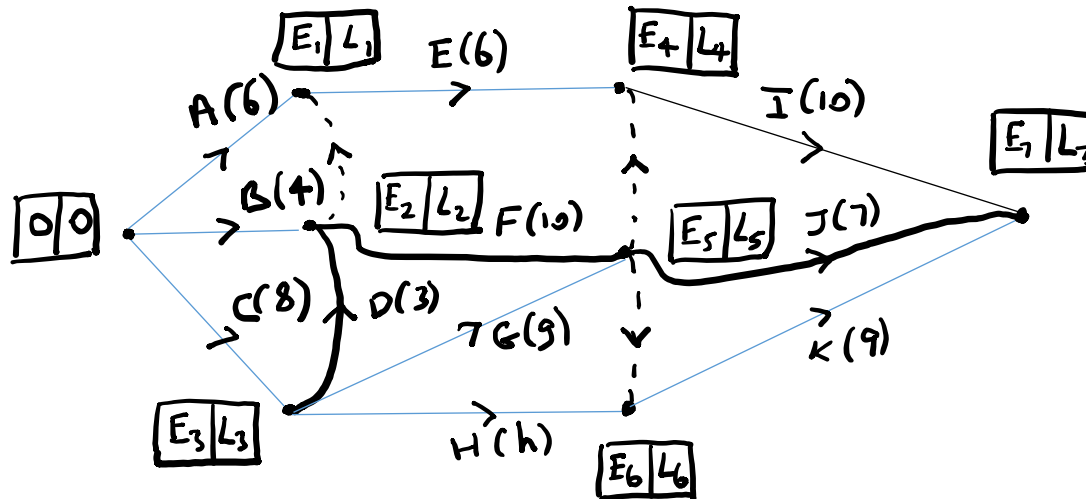
Solution



(5) Create an activity-on-arc network corresponding to the following precedence table, with earliest and latest event times, given that the only critical activities are C, D, F and I.

Activity	Immediate predecessors	Duration (hours)
A	–	6
B	–	4
C	–	8
D	C	3
E	A, B, D	6
F	B, D	10
G	C	
H	C	
I	E, F, G	10
J	F, G	7
K	F, G, H	9

Solution



Denoting the duration of activity A by a etc:

$$E_3 = 0 + 8 = 8; E_2 = \max(E_3 + 3, 0 + b) = \max(11, 4) = 11$$

$$E_1 = \max(E_2 + 0, 0 + a) = \max(11, 6) = 11$$

As C is critical, $L_3 = E_3 = 8$

As D is critical, $L_2 = E_2 = 11$

As F is critical, $E_5 = E_2 + f = 11 + 10 = 21$, and $L_5 = E_5 = 21$

$$E_4 = \max(E_1 + e, E_5 + 0) = \max(11 + 6, 21) = 21$$

As I is critical, $L_4 = E_4 = 21$, $E_7 = E_4 + i = 21 + 10 = 31$

and $L_7 = E_7 = 31$

$$L_6 = L_7 - k = 31 - 9 = 22$$

As K is not critical, and $L_6 + k = L_7$, it follows that $E_6 < L_6$;

ie $E_6 < 22$

$$E_6 = \max(E_3 + h, E_5 + 0) = \max(8 + h, 21)$$

So $E_6 \geq 21$, and hence (as $E_6 < 22$) $E_6 = 21$

