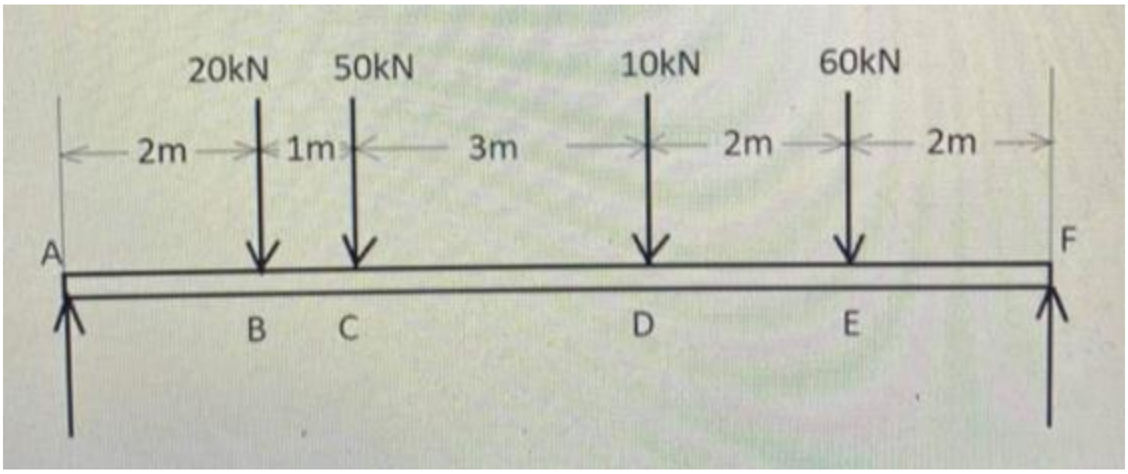
Bending Moment & Shearing Force (5 pages; 25/3/21)

Example



$M(A)$: $1000\{-20(2) - 50(3) - 10(6) - 60(8)\} + R_F(10) = 0$

$\Rightarrow R_F = \frac{730000}{10} = 73000$; ie $73\ kN$

Resolving vertically, $R_A + R_F = 1000(20 + 50 + 10 + 60)$

$= 140000$

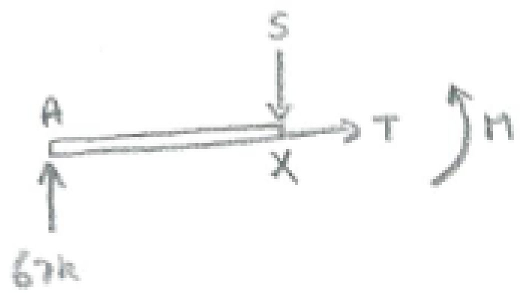
$\Rightarrow R_A = 140000 - 73000 = 67000$; ie $67\ kN$

There will be internal forces and torques in the rod that prevent it from being pulled apart or rotated, and these will vary as we move along the rod.

Consider the segment of the rod AX, where X is at a distance x from A. Suppose initially that $x < 2$.

The diagram below shows the forces and moment (or torque) that would need to be applied at X, were AX to be separated from XF, in order to replicate the effect of the internal forces and torque.

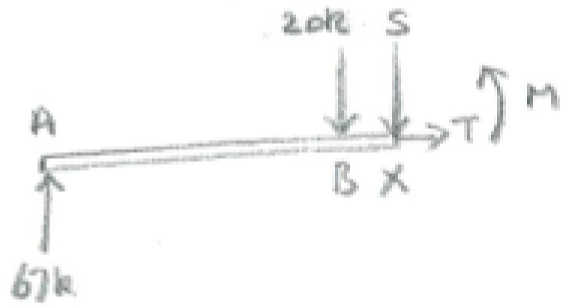
The forces consist of a tension T (or compression, if negative) and a 'shearing force' S, and the torque is referred to as a 'bending moment'.



It may help to compare AX with a door hinged at X, with the door being pushed (gently) at A, but unable to open because of a rusty hinge at X. The hinge is providing a force to counteract the force at A (ie the forces are in equilibrium), and also a moment M to create rotational equilibrium.

The forces and moment at X are determined by resolving the forces on AX, and taking moments about A.

Thus, $T = 0$, $S = 67000$ and $M - Sx = 0$, so that $M = 67000x$

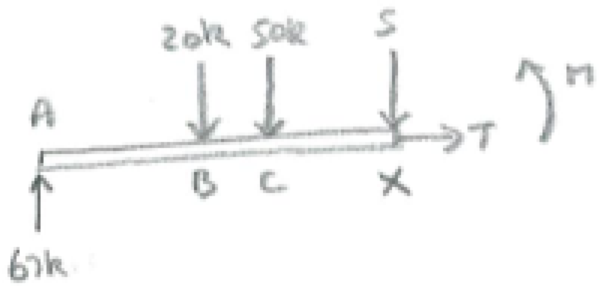


When $2 < x < 3$ (see diagram above), $T = 0$ again,

$S + 20000 = 67000; S = 47000$,

and $M - 20000(2) - Sx = 0$, so that $M = 40000 + 47000x$

[As a check, note that when $x = 2$, $40000 + 47000x = 67000x$]



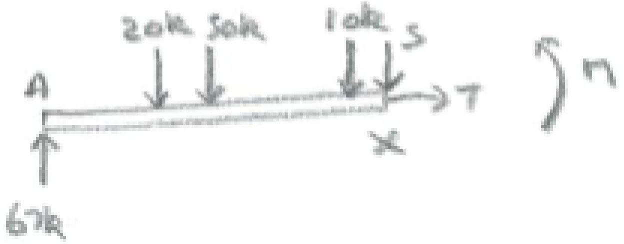
When $3 < x < 6$ (see diagram above),

$S + 20000 + 50000 = 67000; S = -3000$,

and $M - 20000(2) - 50000(3) - Sx = 0$,

so that $M = 190000 - 3000x$

[When $x = 3$, $190000 - 3000x = 40000 + 47000x$]



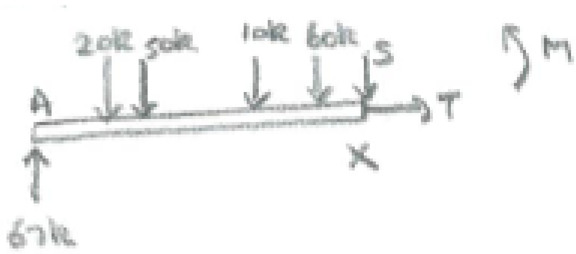
When $6 < x < 8$ (see diagram above),

$S + 20000 + 50000 + 10000 = 67000; S = -13000$,

and $M - 20000(2) - 50000(3) - 10000(6) - Sx = 0$,

so that $M = 250000 - 13000x$

[When $x = 6$, $250000 - 13000x = 190000 - 3000x$]



When $8 < x < 10$ (see diagram above),

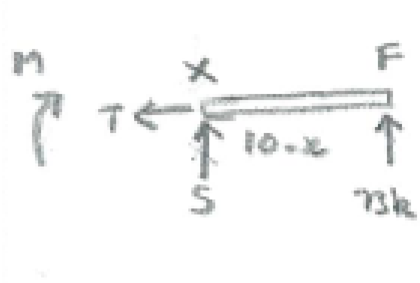
$S + 20000 + 50000 + 10000 + 60000 = 67000$;

$S = -73000$, and

$M - 20000(2) - 50000(3) - 10000(6) - 60000(8) - Sx = 0$,

so that $M = 730000 - 73000x$

[When $x = 8, 730000 - 73000x = 250000 - 13000x$]



As a final check, for $8 < x < 10$, we can determine S and M for XF (see diagram above), noting that the directions are reversed, by Newton's 3rd law (ie the forces applied to AX by XF are equal and opposite to the forces applied to XF by AX.

Thus, $S = -73000$,

and (taking moments about F), $-M - S(10 - x) = 0$,

so that $M = 730000 - 73000x$, as before.

Summary

	Shearing force	Bending moment
$x < 2$	67000	$67000x$
$2 < x < 3$	47000	$40000 + 47000x$
$3 < x < 6$	-3000	$190000 - 3000x$
$6 < x < 8$	-13000	$250000 - 13000x$
$8 < x < 10$	-73000	$730000 - 73000x$

This can be represented graphically as shown below.