

Arithmetic Sequences & Series - Exercises

(2 pages; 22/3/20)

Key to difficulty:

* easier

** moderate

*** harder

(1*) If teams A, B, C, D & E in some sporting competition have to play each other once, how many games are there in total?

(2**) Extend (1) to find a formula for $1 + 2 + 3 + \dots + n$

[See **Solutions** for Ideas]

(3*) For each of the following arithmetic sequences, find an expression for a_k :

(a) in the form $a_k = p + q(k - 1)$

(b) in the form $a_k = mk + c$

(c) in the form $a_k = a_{k-1} + t ; a_1 = u \quad (k \geq 2)$

(where p, q, m, c, t & u are to be found)

(i) 4, 7, 10, 13, 16, ...

(ii) -2, -1, 0, 1, 2, ...

(iii) 8, 6, 4, 2, 0, ...

(4*) If $a_3 = 7$ and $a_{10} = 42$ are terms in an arithmetic sequence, find an expression for a_k .

(5**) Find

(i) $\sum_{k=1}^{20} (2k + 3)$

(ii) $\sum_{k=1}^{40} (10 - 4k)$

(6**) Solve the equation $\sum_{k=1}^n (100 - 5k) = 0$

(7*) For each of the arithmetic sequences in (5), find $\sum_{k=1}^{100} a_k$

(8*) If I pay £50 into a bank account, then £60 a year later, followed by £70 the following year, and so on, increasing by £10 each year,

(i) How long will it take for the amount I pay in each year to reach £200?

(ii) How long will it take for the amount in the bank account to reach £1000?

(9**) For an arithmetic sequence with 1st term a and common difference d , show that the sum of the 1st n terms is

$$\frac{n}{2} [2a + (n - 1)d] \text{ by starting with } \sum_{k=1}^n [a + (k - 1)d]$$