Approximations to the Binomial and Poisson Distributions

(4 pages; 30/1/17)

(1) Overview of Approximations

(i) There is a one-way direction for approximations:

Binomial → Poisson → Normal

Provided the necessary conditions are satisfied,

the Binomial distribution can be approximated by the Poisson distribution, or by the Normal distribution;

the Poisson distribution can be approximated by the Normal distribution;

but the Poisson is never approximated by the Binomial, and the Normal is never approximated by either the Binomial or the Poisson.

(ii) A discrete variable may sometimes be treated as approximately Normal (ie even if it is not Binomial or Poisson)

(2) Binomial approximated by Poisson

$B(n,p) \to Po(np)$

Necessary conditions:

- n is large and p is small

$(\Rightarrow q = 1 - p \approx 1\ and\ so\ np \approx npq;\ \ Var = Mean\ for\ Poisson)$

If $np > 10$, then the Binomial would usually be approximated by the Normal instead (see later on). (Poisson tables only go up to λ =10 usually.)

Notes:

(i) Use Binomial (rather than Poisson approximation), if possible.

(ii) If p is close to 1, use the 'complementary' random variable: counting failures (with small probability) as successes.

(iii) Often the value of np will be decisive, when choosing between an approximation by the Poisson or by the Normal distibution (discussed later on).

(3) Discrete variable (need not be Binomial or Poisson) being treated as approximately Normal

Ensure that the step in the discrete values (often 1) is relatively small, compared with the standard deviation.

Continuity Correction

There is a correspondence between the discrete values and the continuous values of the Normal distribution,

so that if X is discrete and Y is the Normal approximation,

then $10 \leq X \leq 20 \leftrightarrow 9.5 \leq Y < 20.5$,

as the Y interval must include all values that round to 10 or 20; though we can also write $9.5 < Y < 20.5$, as $P(Y = 9.5) = 0$ for a continuous variable

Examples

(i) $X \geq 20 \leftrightarrow Y > 19.5$

(ii) $X < 10 \equiv X \leq 9 \leftrightarrow Y < 9.5$

(4) Binomial approximated by Normal

$B(n, p) \to N(np, npq)$

Necessary conditions:

- n is not too small

- p is not too close to 0 or 1 (to ensure symmetry)

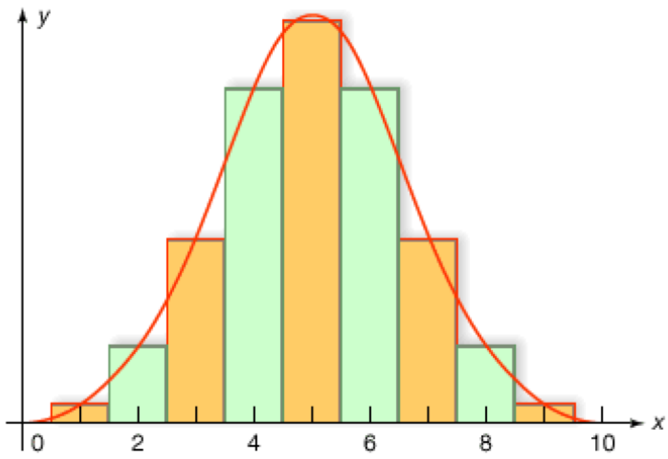
- the step in the discrete values (often 1) is relatively small, compared with the standard deviation

The closer that p is to 0 or 1, the larger n has to be, for an approximation to be possible.

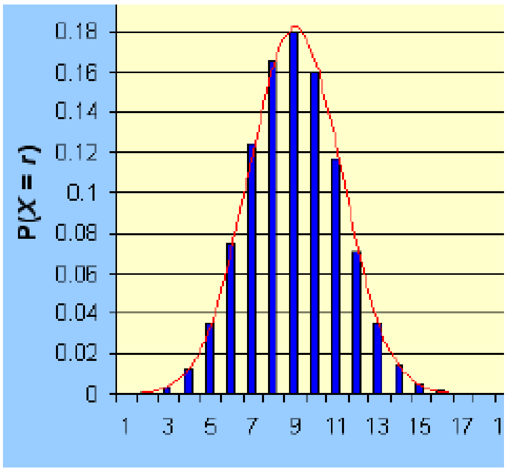
There will be a good approximation if $n > 20$ and p is close to 0.5

Note: If $np \leq 10$, the Poisson approximation can be used instead (if conditions are satisfied; ie large n and small p)

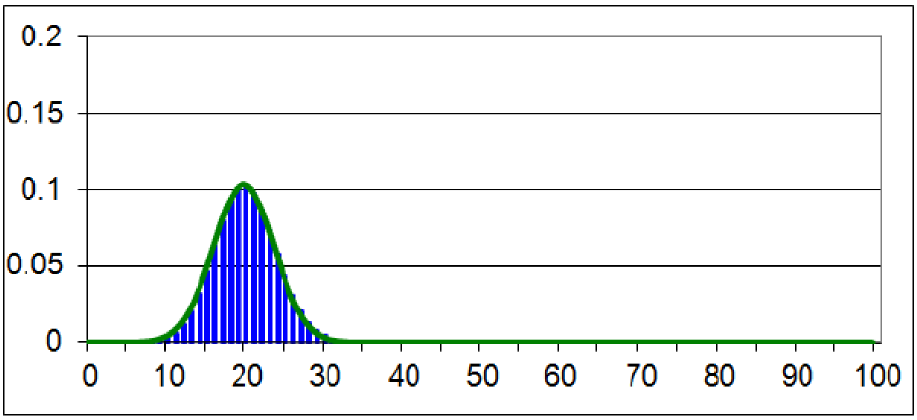
A continuity correction will be required (as described above).



X = Binomial (10, ½)   X = Normal (5, 2.5)



$B(20,0.4)$



$B(80,0.25)$

(5) Poisson approximated by Normal

$Po(\lambda) \rightarrow N(\lambda, \lambda)$

Necessary conditions:

- λ sufficiently large (> 10)

(ensures that 3 standard deviations from the mean ($10 - 3\sqrt{10}$) doesn't stray into negative numbers)

A continuity correction will be required (as described above).