

Algorithms - Exercises (Sol'ns) (5 pages; 3/2/20)

(1**) (i) By performing traces, or otherwise, establish what the following algorithm achieves.

10 Input N

20 $e = 0.0001$

30 $L = 1000$

40 $F = 0$

50 $x = \frac{N}{2}$

60 $y = \frac{N}{x}$

70 $z = x$

80 $x = \frac{x+y}{2}$

90 If $|x - z| < e$ Then Goto 130

100 $F = F + 1$

110 If $F > L$ Then Goto 140

120 Goto 60

130 Print x

140 Print "End"

150 END

(ii) What roles do e , F and L play in the algorithm?

Solution

(i) The algorithm finds the square root of N .

(ii) e determines when the successive estimates are close enough together

F counts the number of iterations

L is the limit imposed on the number of iterations

Note that e and L are 'hard-coded', to avoid having too many inputs, but that their values are stored at the start of the program, so that changes can easily be made.

(2**) Use the Bubble Sort algorithm to sort the following items into increasing order.

7 45 13 27 6 19 44 15 21

Solution

Original list: 7 45 13 27 6 19 44 15 21

1st Pass: 7 13 27 6 19 44 15 21 45

2nd Pass: 7 13 6 19 27 15 21 44 45

3rd Pass: 7 6 13 19 15 21 27 44 45

4th Pass: 7 6 13 15 19 21 27 44 45

5th Pass: 7 6 13 15 19 21 27 44 45

[The 5th pass is necessary, in order to establish that no further swaps are needed.]

(3**) Use the Quick Sort algorithm to sort the following items into increasing order.

7 45 13 27 6 19 44 15 21

Solution

Step 1: Select the middle item (or the right-hand of the two middle items) as the pivot [in brackets here, but circled when handwritten]

7 45 13 27 (6) 19 44 15 21

Step 2: Pivot about 6: the remaining items (ie on either side of the 6) that are less than (or equal to) 6 go to the left of (6), without changing their order; similarly, items that are greater than 6 go to the right. The pivot is underlined, to indicate that its position in the list is fixed. Also, the next pivot(s) are labelled.

6 7 45 13 27 (19) 44 15 21

Step 3: The process is repeated until all items are fixed:

6 7 (13) 15 19 45 27 (44) 21

[Here there are two 'sub-lists': 7 13 15 and 45 27 44 21; both of which produce a pivot.]

6 (7) 13 (15) 19 27 (21) 44 (45)

[Strictly speaking, 7, 15 and 45 are pivots, even though they are in sub-lists of 1 item each.]

6 7 13 15 19 21 (27) 44 45

6 7 13 15 19 21 27 44 45

Note: Sometimes the first item is chosen as the pivot. Also the method of labelling the pivot varies. It is recommended to describe what is being done, and to define any labelling (so that it is clear that the pivoting process occurs in the line after the pivot has been identified).

(4**) A library needs to store away some of its books (to make way for more computers). Each of the storage boxes can contain 100 books. The shelves to be stored have the following contents, and each shelf has to be stored in a single box.

Photography: 7 books

Fiction: 45 books

Pastimes: 13 books

History: 27 books

Art: 6 books

Computers: 19 books

Biographies: 44 books

Self-help: 15 books

Science : 21 books

- (i) Apply the First-fit algorithm.
- (ii) Apply the First-fit Decreasing algorithm.
- (iii) Apply the Full bins method.

Solution

(i) Box 1: 7, 45, 13, 27, 6 (98)

Box 2: 19, 44, 15, 21 (99)

(ii) In decreasing order: 45, 44, 27, 21, 19, 15, 13, 7, 6

Box 1: 45, 44, 7 (96)

Box 2: 27, 21, 19, 15, 13 (95)

Box 3: 6

[So the First-fit Decreasing algorithm can, in some cases, be less effective than the First-fit algorithm.]

(iii) For example,

Box 1: 7, 45, 15, 27, 6 (100)

Box 2 : 13, 19, 44, 21 (97)