$y^2 = f(x)$ (2 pages; 2/6/23)

(1) The graph is undefined where $f(x) < 0$.

(2) There will be two branches of the graph: $y = \pm\sqrt{f(x)}$ (so that the graph is symmetric about the x-axis).

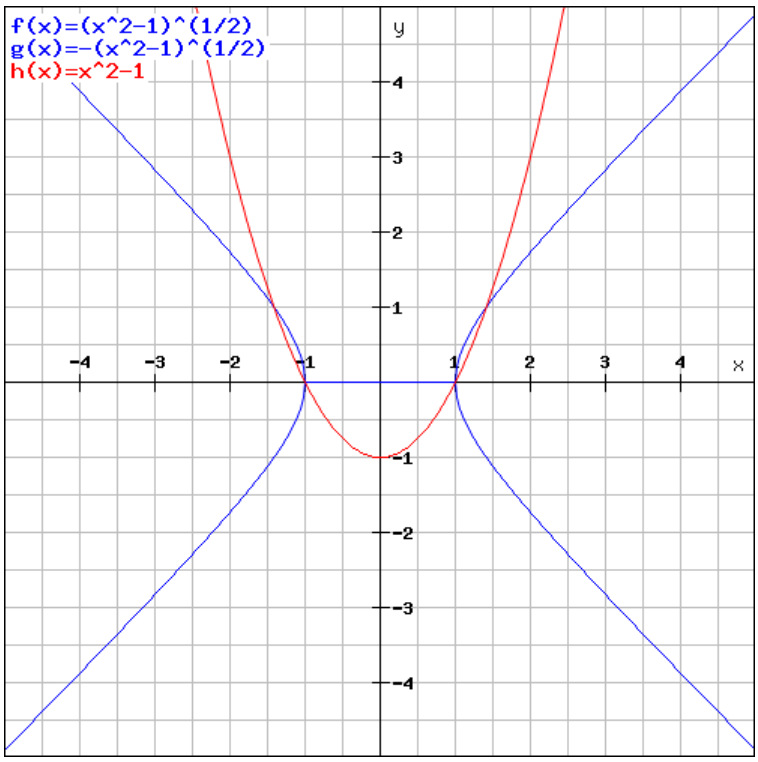
(3) $y^2 = y$ when $y = 0$ or 1; so these are the x- values where $y^2 = f(x)$ crosses $y = f(x)$

(4) Differentiating wrt x, $2y\frac{dy}{dx} = f'(x)$ (A)

Considering the branch for which $y \geq 0$, the gradient of $y^2 = f(x)$ (or $y = \sqrt{f(x)}$); ie $\frac{dy}{dx}$ has the same sign as the gradient of $y = f(x)$; ie $f'(x)$

Also, the top branch of $y^2 = f(x)$ has turning points when $y = f(x)$ has turning points.

(5) Provided $f'(x) \neq 0$, (A) $\Rightarrow \frac{dy}{dx} = \infty$ (ie the graph is vertical) when $y = 0$



$y = x^2 - 1$ and $y^2 = x^2 - 1$