Vectors Q9 (3/7/23)

Find the cartesian form of the plane

$$\underline{r} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

Solution

$$\underline{n} = \begin{pmatrix}1\\4\\4\end{pmatrix} \times \begin{pmatrix}2\\3\\1\end{pmatrix} = \begin{vmatrix}\underline{i} & 1 & 2\\ \underline{j} & 4 & 3\\ \underline{k} & 4 & 1\end{vmatrix} = \begin{pmatrix}-8\\7\\-5\end{pmatrix}$$

$$\left(\begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}0\\-2\\-1\end{pmatrix}\right).\begin{pmatrix}-8\\7\\-5\end{pmatrix} = 0 \Rightarrow -8x + 7(y+2) - 5(z+1) = 0$$

$$\Rightarrow -8x + 7y - 5z = -9 \text{ or } 8x - 7y + 5z = 9$$

Alternative version (once $\underline{n}$ has been found)

Let plane be $-8x + 7y - 5z = p$

As $\begin{pmatrix}0\\-2\\-1\end{pmatrix}$ lies in the plane, $-8(0) + 7(-2) - 5(-1) = p$;

so $p = -9$ etc

Alternative method

Eliminate s & t from the 3 simultaneous equations.