

Vectors Q8 (3/7/23)

Find the acute angle between the line $\frac{x-4}{-3} = \frac{y+2}{5}, z = -2$ and the plane $2x - z = 7$.

Solution

The angle between the direction vector of the line and the normal to the plane is given by

$$\begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \sqrt{(-3)^2 + 5^2 + 0^2} \sqrt{2^2 + 0^2 + (-1)^2} \cos\theta$$

$$\Rightarrow -6 = \sqrt{34}\sqrt{5} \cos\theta \Rightarrow \cos\theta = -\frac{6}{\sqrt{170}} \Rightarrow \theta = 117.399 = 117.4^\circ$$

(1dp) [this is usually the preferred degree of accuracy for an angle given in degrees]

This means that the acute angle between these two directions is

$180 - 117.4 = 62.6$, and the angle between the plane itself and the line is therefore $90 - 62.6 = 27.4^\circ$