Vectors Q6 (3/7/23)

Find a vector equation of the line that passes through the point (1,2) and is perpendicular to the line $\underline{r} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

Solution

Method 1

The gradient of the given line is $\frac{-1}{4}$, so that the gradient of the perpendicular line is 4.

Then a vector equation of the required line is

$$\underline{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

Method 2 (much longer, but good practice!)

Let P be the intersection of the given line (L, say) and the perpendicular line through Q(1,2). Then P can be represented as

$\begin{pmatrix} 3 + 4\lambda \\ 4 - \lambda \end{pmatrix}$, for some λ to be determined.

Then, as L is perpendicular to QP, $\begin{pmatrix} 4 \\ -1 \end{pmatrix} . \begin{pmatrix} 3 + 4\lambda - 1 \\ 4 - \lambda - 2 \end{pmatrix} = 0$

[noting that $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ is the direction vector of L; not to be confused with $\begin{pmatrix} 3 + 4\lambda \\ 4 - \lambda \end{pmatrix}$, which the position vector of a point on L]

so that $4(2 + 4\lambda) - (2 - \lambda) = 0$,

and hence $17\lambda + 6 = 0$, and $\lambda = -\frac{6}{17}$

Thus P is $\begin{pmatrix} 3 + 4\left(-\frac{6}{17}\right) \\ 4 - \left(-\frac{6}{17}\right) \end{pmatrix} = \frac{1}{17}\begin{pmatrix} 27 \\ 74 \end{pmatrix}$

And a vector equation of the line through P and Q is

$$\underline{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda[\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \frac{1}{17}\begin{pmatrix} 27 \\ 74 \end{pmatrix}]$$

or $\underline{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \frac{\lambda}{17}\begin{pmatrix} 17 - 27 \\ 34 - 74 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \frac{\lambda}{17}\begin{pmatrix} -10 \\ -40 \end{pmatrix}$

or $\underline{r} = \begin{pmatrix}1\\2\end{pmatrix} + \mu\begin{pmatrix}1\\4\end{pmatrix}$