# Vectors Q4 (3/7/23)

Find $c, a$ & $b$ such that $\begin{pmatrix} 2 \\ 3 \\ c \end{pmatrix} = a\begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + b\begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$

[ie such that the 3 vectors are not linearly independent]

**Solution**

As the position vector $\begin{pmatrix}2\\3\\c\end{pmatrix}$ is in the plane containing the Origin and the position vectors $\begin{pmatrix}-1\\0\\3\end{pmatrix}$ & $\begin{pmatrix}0\\2\\4\end{pmatrix}$, it follows that $\begin{pmatrix}2\\3\\c\end{pmatrix}$ is perpendicular to the normal to that plane; ie perpendicular to

$$\begin{pmatrix}-1\\0\\3\end{pmatrix}\times\begin{pmatrix}0\\2\\4\end{pmatrix}=\begin{vmatrix}\underline{i} & -1 & 0\\ \underline{j} & 0 & 2\\ \underline{k} & 3 & 4\end{vmatrix}$$ ; so that

$$\begin{pmatrix}2\\3\\c\end{pmatrix}.\begin{vmatrix}\underline{i} & -1 & 0\\ \underline{j} & 0 & 2\\ \underline{k} & 3 & 4\end{vmatrix}=0$$ , and thus $$\begin{vmatrix}2 & -1 & 0\\ 3 & 0 & 2\\ c & 3 & 4\end{vmatrix}=0$$

**Alternative Approach 1**

The 3 vectors form a parallelepiped of zero volume, so that the scalar triple product of the vectors is zero.]

**Alternative Approach 2**

The required relation can be written as

$$\begin{pmatrix}2\\3\\c\end{pmatrix}-a\begin{pmatrix}-1\\0\\3\end{pmatrix}-b\begin{pmatrix}0\\2\\4\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix},$$

which implies a solution of $x\begin{pmatrix}2\\3\\c\end{pmatrix}+y\begin{pmatrix}-1\\0\\3\end{pmatrix}+z\begin{pmatrix}0\\2\\4\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$

other than $x=y=z=0$,

and for there to be more than one solution to this matrix equation, we require the determinant to be zero.

Then $\begin{vmatrix} 2 & -1 & 0 \\ 3 & 0 & 2 \\ c & 3 & 4 \end{vmatrix} = 0 \Rightarrow 2(-6) - (-1)(12 - 2c) = 0$

$\Rightarrow c = 0$

$\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = a \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + b \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$

$\Rightarrow 2 = -a$

$3 = 2b$

& $0 = 3a + 4b$

so that $a = -2$ & $b = \frac{3}{2}$