Vectors Q2 (3/7/23)

Find the shortest distance between the lines

$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+3}{2}$ and $\frac{x+5}{7} = \frac{y}{1} = \frac{z-1}{3}$

Solution

[Note: The following method is probably the quickest way of finding the shortest distance, but doesn't generate the points on the lines that are closest.]

Shortest distance, $D = \left|\frac{(\underline{d}_1 \times \underline{d}_2).(\underline{a}_1 - \underline{a}_2)}{|\underline{d}_1 \times \underline{d}_2|}\right|$

where $\underline{d}_1 = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$, $\underline{d}_2 = \begin{pmatrix} 7 \\ 1 \\ 3 \end{pmatrix}$, $\underline{a}_1 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$, $\underline{a}_2 = \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix}$

Then $\underline{d}_1 \times \underline{d}_2 = \begin{vmatrix} \underline{i} & 4 & 7 \\ \underline{j} & 3 & 1 \\ \underline{k} & 2 & 3 \end{vmatrix} = \begin{pmatrix} 7 \\ 2 \\ -17 \end{pmatrix}$ and $\underline{a}_1 - \underline{a}_2 = \begin{pmatrix} 7 \\ 1 \\ -4 \end{pmatrix}$

so that $D = \left|\frac{7(7)+2(1)+(-17)(-4)}{\sqrt{7^2+2^2+(-17)^2}}\right| = \left|\frac{119}{\sqrt{342}}\right| = \frac{119}{\sqrt{342}}$