# Vectors Q1 (3/7/23)

(i) Show that the line $\underline{r} = \underline{a} + t\underline{b}$ and the plane $\underline{r}.\underline{n} = d$ intersect at the point $\underline{r} = \underline{a} + \left(\frac{d - \underline{a}.\underline{n}}{\underline{b}.\underline{n}}\right)\underline{b}$

(ii) Find the intersection of the line $\underline{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and the plane $\underline{r}.\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = -2$

(iii) Find the angle between the line and the plane in (ii).

**Solution**

(i) $(\underline{a} + t\underline{b}).\underline{n} = d \Rightarrow \underline{a}.\underline{n} + t\underline{b}.\underline{n} = d$

$\Rightarrow t = \frac{d-\underline{a}.\underline{n}}{\underline{b}.\underline{n}}$

$\Rightarrow \underline{r} = \underline{a} + (\frac{d-\underline{a}.\underline{n}}{\underline{b}.\underline{n}})\underline{b}$

(ii) Applying the result in (i):

$\underline{a}.\underline{n} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = -1$

and $\underline{b}.\underline{n} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}.\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = -1$

so that $\underline{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + (\frac{-2-[-1]}{-1})\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}.\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \sqrt{1+4+0}\,.\sqrt{1+1+0}\,cos\theta$

$\Rightarrow cos\theta = \frac{1-2+0}{\sqrt{5}.\sqrt{2}} = -\frac{1}{\sqrt{10}}$

$\Rightarrow \theta = 108.43495°$

The required angle is:

$90 - (180 - 108.43495) = 18.43495 = 18.4°$ (1dp)