Vectors Q15 (3/7/23)

(i) Find a vector that is perpendicular to both $\begin{pmatrix} 7 \\ 0 \\ -10 \end{pmatrix}$ & $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

(ii) Use (i) to find the plane that passes through the points with position vectors $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 8 \\ 2 \\ -7 \end{pmatrix}$ & $\begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}$

Solution

(i) **Method 1**

$$\begin{pmatrix} 7 \\ 0 \\ -10 \end{pmatrix} \times \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} = \begin{vmatrix} \underline{i} & 7 & 1 \\ \underline{j} & 0 & 3 \\ \underline{k} & -10 & -1 \end{vmatrix} = \begin{pmatrix} 30 \\ -3 \\ 21 \end{pmatrix}$$

[From the theory of the vector product,] this is perpendicular to the given vectors (as is $\frac{1}{3}\begin{pmatrix} 30 \\ -3 \\ 21 \end{pmatrix} = \begin{pmatrix} 10 \\ -1 \\ 7 \end{pmatrix}$).

Method 2

Let the required vector be $\begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$

Then $\begin{pmatrix} 7 \\ 0 \\ -10 \end{pmatrix} . \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} = 0$, so that $7 - 10b = 0$ & $b = \frac{7}{10}$

And $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} . \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} = 0$, so that $1 + 3a - b = 0$,

and $a = \frac{1}{3}\left(\frac{7}{10} - 1\right) = -\frac{1}{10}$

Thus (multiplying by 10) a suitable vector is $\begin{pmatrix} 10 \\ -1 \\ 7 \end{pmatrix}$.

(ii) Let $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 8 \\ 2 \\ -7 \end{pmatrix}$ & $\begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}$ represent the points A, B & C, respectively.

Then $\overrightarrow{AB} = \begin{pmatrix} 7 \\ 0 \\ -10 \end{pmatrix}$ & $\overrightarrow{AC} = \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} = -\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

From (i), a vector that is perpendicular to $\overrightarrow{AB}$ & $\overrightarrow{AC}$ (and therefore a normal to the plane) is $\begin{pmatrix} 10 \\ -1 \\ 7 \end{pmatrix}$.

So the equation of the plane is

$$\underline{r}\,.\begin{pmatrix} 10 \\ -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.\begin{pmatrix} 10 \\ -1 \\ 7 \end{pmatrix} = 10 - 2 + 21 = 29$$

or (in cartesian form) $10x - y + 7z = 29$ [as $\underline{r} \equiv \begin{pmatrix} x \\ y \\ z \end{pmatrix}$]