Vectors Q13 (3/7/23)

(i) Given that the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$ is $\frac{|d-\underline{p}.\underline{n}|}{|\underline{n}|}$, what is the significance of $\frac{d}{|\underline{n}|}$ if $d > 0$?

(ii) Find the equation of the plane that is parallel to $\underline{r}.\underline{n} = d$ and contains the point $\underline{p}$.

(iii) Hence deduce the formula for the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$

Solution

(i) $\frac{d}{|\underline{n}|}$ is the distance of the plane $\underline{r}.\underline{n} = d$ from the Origin,

when $d > 0$

(ii) $\underline{r}.\underline{n} = \underline{p}.\underline{n}$

(iii) The shortest distance from the plane $\underline{r}.\underline{n} = d$ to the Origin is $\frac{d}{|\underline{n}|}$.

The plane parallel to $\underline{r}.\underline{n} = d$, containing $\underline{p}$ has equation

$\underline{r}.\underline{n} = \underline{p}.\underline{n}$, and its shortest distance from the Origin is $\frac{\underline{p}.\underline{n}}{|\underline{n}|}$

Hence the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$

is $\left|\frac{\underline{p}.\underline{n}}{|\underline{n}|} - \frac{d}{|\underline{n}|}\right| = \frac{|d-\underline{p}.\underline{n}|}{|\underline{n}|}$