Vectors Q11 (3/7/23)

Find the plane containing the points
$(2, -1,4), (-3,4,2)$ and $(1,0,5)$, in Cartesian form

Solution

Method 1

In parametric form, it is:

$$\underline{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \lambda\left[\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}\right] + \mu[\begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}]$$

or $\underline{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \lambda\begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix} + \mu\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$x = 2 - 5\lambda - \mu \quad (1)$

$y = -1 + 5\lambda + \mu \quad (2)$

$z = 4 - 2\lambda + \mu \quad (3)$

Eliminating λ & μ: $(1) + (2) \Rightarrow x + y = 1$

Method 2

The normal to the plane is perpendicular to both

$\begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$; eg $\begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{vmatrix} \underline{i} & -5 & -1 \\ \underline{j} & 5 & 1 \\ \underline{k} & -2 & 1 \end{vmatrix}$

$= \begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$,

so that eq'n is $\underline{r}.\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}.\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ or $x + y = 1$