# Vectors: Exercises - Overview (3/7/23)

## Lines

### Q5

Given that the line $\underline{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ can also be written as

$\begin{pmatrix} 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, find $\mu$ in terms of $\lambda$

### Q6

Find a vector equation of the line that passes through the point (1,2) and is perpendicular to the line $\underline{r} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

## Lines & Planes

### Q1

(i) Show that the line $\underline{r} = \underline{a} + t\underline{b}$ and the plane $\underline{r}.\underline{n} = d$ intersect at the point $\underline{r} = \underline{a} + \left(\frac{d - \underline{a}.\underline{n}}{\underline{b}.\underline{n}}\right)\underline{b}$

(ii) Find the intersection of the line $\underline{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and the plane $\underline{r}.\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = -2$

(iii) Find the angle between the line and the plane in (ii).

## Q8

Find the acute angle between the line $\frac{x-4}{-3}=\frac{y+2}{5}, z=-2$ and the plane $2x-z=7$.

## Q10

(i)(a) Find the acute angle between the line $\frac{x}{2}=\frac{y+1}{-3}=\frac{z-2}{1}$ and the plane $x+y-2z=5$

(b) Show that the same angle is obtained if the line is written in the form

$\frac{x}{-2}=\frac{y+1}{3}=\frac{z-2}{-1}$ (ie without rearranging into the form in (a))

(ii)(a) Find the acute angle between the planes $x+4y-3z=7$

and $x-y+4z=2$

(b) Find the acute angle between the planes $x+4y-3z=7$ and $-x+y-4z=2$ (again, without rearranging the equation)

## Q17

Find the line that is the reflection of the line $\frac{x-2}{3}=\frac{y}{4}=\frac{z+1}{1}$ in the plane $x-2y+z=4$

## Q20

Find the reflection of the line $\frac{x-2}{3}=\frac{y+4}{1}; z=3$ in the plane $y=4$

# Planes

## Q7

Find the angle between the planes $x = 2$ and $y + 2z = 3$

## Q9

Find the cartesian form of the plane

$$\underline{r} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

## Q11

Find the plane containing the points
$(2, -1,4), (-3,4,2)$ and $(1,0,5)$, in Cartesian form

## Q15

(i) Find a vector that is perpendicular to both $\begin{pmatrix} 7 \\ 0 \\ -10 \end{pmatrix}$ & $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

(ii) Use (i) to find the plane that passes through the points with position vectors $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ , $\begin{pmatrix} 8 \\ 2 \\ -7 \end{pmatrix}$ & $\begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}$

## Q25

(i) Find the plane containing the points
$(3,0,-1), (5,2,-3)$ and $(4,2,4)$, in parametric form

(ii) Hence find the equation of the plane in Cartesian form.

## Shortest Distance

### Q2

Find the shortest distance between the lines

$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+3}{2}$ and $\frac{x+5}{7} = \frac{y}{1} = \frac{z-1}{3}$

### Q3

Find the shortest distance between the lines

$\frac{x-1}{2} = \frac{y+3}{5} = \frac{z-2}{3}$ and $\frac{x}{1} = \frac{y-4}{2} = \frac{z+1}{2}$ , identifying the points on the lines at which this shortest distance occurs.

### Q12

(i) Find the intersection of the line $\underline{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ and the plane $3x + y + 4z = 77$

(ii) Find the shortest distance from the point $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ to the plane

$3x + y + 4z = 77$

**Q13**

(i) Given that the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$ is $\frac{|d-\underline{p}.\underline{n}|}{|\underline{n}|}$, what is the significance of $\frac{d}{|\underline{n}|}$ if $d > 0$?

(ii) Find the equation of the plane that is parallel to $\underline{r}.\underline{n} = d$ and contains the point $\underline{p}$.

(iii) Hence deduce the formula for the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$

**Q16**

Find the shortest distance between the point $(4, -2, 3)$ and the line $\underline{r} = \begin{pmatrix} 7 \\ 5 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -6 \\ 4 \end{pmatrix}$, leaving the answer in surd form.

**Q18**

Find the distance between the lines $\frac{x+1}{1} = \frac{y+2}{2}; z = 4$ and $\frac{x+3}{1} = \frac{y-6}{2}$ ; $z = 7$ , leaving your answer in exact form.

**Q19**

(i) Show the lines $\frac{x-1}{2} = \frac{y+3}{5} = \frac{z-2}{3}$ and $\frac{x}{1} = \frac{y-4}{2} = \frac{z+1}{2}$ are skew.

(ii) Find the shortest distance between the lines and identify the points on the lines at which this shortest distance occurs.

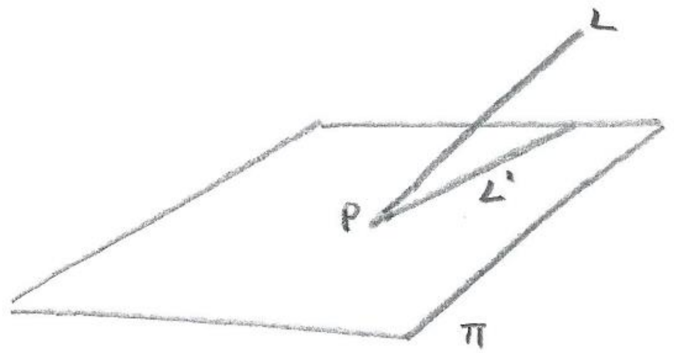
**Q22**

Show that the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$ is $\frac{|d-\underline{p}.\underline{n}|}{|\underline{n}|}$

**Q23**

Given the plane Π: $3x + 2y - z = 6$ and the line

$L: \underline{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, let L' be the projection of L onto Π




(i) Find the point of intersection (P) of Π & L

(ii) Find the angle between Π & L

(iii) Find a vector that is parallel to Π and perpendicular to L

(iv) Find a vector equation for L'

(v) Find the angle between L and L'

# Vector Product

## Q14

Find the volume of the tetrahedron with corners
$(2, 1, 3), (-1, 5, 0), (4, 4, 7), (8, 2, 2)$

## Q21

Use the vector product to find the area of the triangle with corners A (1,2,3), B (4,5,6) & C (9,8,7)

# Miscellaneous

## Q4

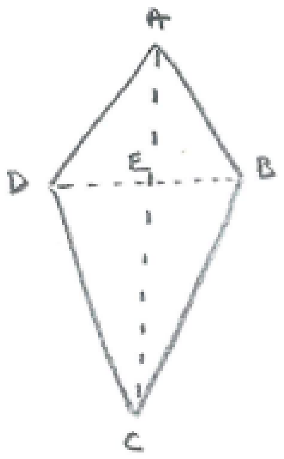
Find $c, a$ & $b$ such that $\begin{pmatrix} 2 \\ 3 \\ c \end{pmatrix} = a\begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + b\begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$

[ie such that the 3 vectors are not linearly independent]

## Q24

In the diagram below, ABCD is a kite. Find $\overrightarrow{OD}$ if $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 4/3 \\ 7 \end{pmatrix}$ ,

$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 4/3 \\ 2 \end{pmatrix}$ & $\overrightarrow{OC} = \begin{pmatrix} 6 \\ 16/3 \\ 2 \end{pmatrix}$

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