Vectors Exercises - Part 1 (4 pages; 3/2/20)

Key to difficulty:

* easier

** moderate

*** harder

(1*) Vector equation of line

Given that the line $\underline{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ can also be written as

$\begin{pmatrix} 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, find μ in terms of λ

(2*) Vector equation of line

Find a vector equation of the line that passes through the point (1,2) and is perpendicular to the line $\underline{r} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

(3**) Scalar product

Show that if $|\underline{a} - \underline{b}| = |\underline{a} + \underline{b}|$, then $\underline{a}$ & $\underline{b}$ are perpendicular.

(4**) Planes

Find the cartesian form of the plane

$$\underline{r} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

(5***) Lines and planes

Find the line that is the reflection of the line $\frac{x-2}{3} = \frac{y}{4} = \frac{z+1}{1}$ in the plane $x - 2y + z = 4$

(6**) Lines and planes

(i)(a) Find the acute angle between the line $\frac{x}{2} = \frac{y+1}{-3} = \frac{z-2}{1}$ and the plane $x + y - 2z = 5$

(b) Show that the same angle is obtained if the line is written in the form

$\frac{x}{-2} = \frac{y+1}{3} = \frac{z-2}{-1}$ (ie without rearranging into the form in (a))

(ii)(a) Find the acute angle between the planes $x + 4y - 3z = 7$ and
$x - y + 4z = 2$

(b) Find the acute angle between the planes $x + 4y - 3z = 7$ and
$-x + y - 4z = 2$ (again, without rearranging the equation)

(7***) Lines and planes

Find the line that is the reflection of the line $\frac{x-2}{3} = \frac{y}{4} = \frac{z+1}{1}$ in the plane $x - 2y + z = 4$

(8***) Lines

Find the distance between the lines $\frac{x+1}{1} = \frac{y+2}{2}; z = 4$ and
$\frac{x+3}{1} = \frac{y-6}{2}; z = 7$, leaving your answer in exact form.

(9***) Lines

(i) Show the lines $\frac{x-1}{2}=\frac{y+3}{5}=\frac{z-2}{3}$ and $\frac{x}{1}=\frac{y-4}{2}=\frac{z+1}{2}$ are skew.

(ii) Find the shortest distance between the lines and identify the points on the lines at which this shortest distance occurs.

(10**) Lines

Given that $A=\begin{pmatrix}1\\2\\3\end{pmatrix}$, $B=\begin{pmatrix}-4\\3\\1\end{pmatrix}$, $C=\begin{pmatrix}0\\1\\2\end{pmatrix}$, $D=\begin{pmatrix}p\\4\\-4\end{pmatrix}$

(i) Write down the equations of the lines AB and CD (both extended)

(ii) Find $\overrightarrow{AB}\times\overrightarrow{CD}$

(iii) For what value of p are the lines AB and CD parallel? (2 methods)

(11**) Planes

Find the plane containing the points
$(2,-1,4)$, $(-3,4,2)$ and $(1,0,5)$, in Cartesian form

(12***) Lines and planes

Find the reflection of the line $\frac{x-2}{3}=\frac{y+4}{1}; z=3$ in the plane $y=4$

(13***) Problem

Use vectors to prove that the mid-points of the sides of any quadrilateral form the vertices of a parallelogram.