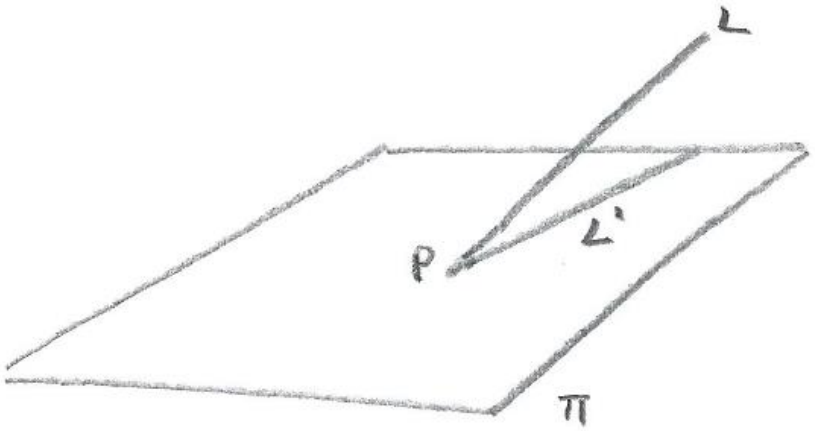
Vectors - Exercises: Lines & Planes (4 pages; 13/8/19)

(1) Find a vector equation of the line that passes through the point (1,2) and is perpendicular to the line $\underline{r} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

(2) Given the plane Π: $3x + 2y - z = 6$ and the line

$L: \underline{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, let L' be the projection of L onto Π



(i) Find the point of intersection (P) of Π & L

(ii) Find the angle between Π & L

(iii) Find a vector that is parallel to Π and perpendicular to L

(iv) Find a vector equation for L'

(v) Find the angle between L and L'

(3) Find the cartesian form of the plane

$$\underline{r} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

(4)(i) Find the intersection of the line $\underline{r} = \underline{a} + t\underline{b}$ and the plane $\underline{r}.\underline{n} = d$

(ii) Find the intersection of the line $\underline{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and the plane $\underline{r}.\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = -2$

(5) Find the line that is the reflection of the line $\frac{\boldsymbol{x-2}}{\boldsymbol{3}} = \frac{\boldsymbol{y}}{\boldsymbol{4}} = \frac{\boldsymbol{z+1}}{\boldsymbol{1}}$ in the plane $\boldsymbol{x - 2y + z = 4}$

(6)(i)(a) Find the acute angle between the line $\frac{\boldsymbol{x}}{\boldsymbol{2}} = \frac{\boldsymbol{y+1}}{\boldsymbol{-3}} = \frac{\boldsymbol{z-2}}{\boldsymbol{1}}$ and the plane

$\boldsymbol{x + y - 2z = 5}$

(b) Show that the same angle is obtained if the line is written in the form

$\frac{\boldsymbol{x}}{\boldsymbol{-2}} = \frac{\boldsymbol{y+1}}{\boldsymbol{3}} = \frac{\boldsymbol{z-2}}{\boldsymbol{-1}}$ (ie without rearranging into the form in (a))

(ii)(a) Find the acute angle between the planes $\boldsymbol{x + 4y - 3z = 7}$ and

$\boldsymbol{x - y + 4z = 2}$

(b) Find the acute angle between the planes $\boldsymbol{x + 4y - 3z = 7}$ and

$\boldsymbol{-x + y - 4z = 2}$ (again, without rearranging the equation)

(7) Find the line that is the reflection of the line $\frac{x-2}{3} = \frac{y}{4} = \frac{z+1}{1}$ in the plane $\boldsymbol{x - 2y + z = 4}$

(8) Find the distance between the lines $\frac{x+1}{1} = \frac{y+2}{2}; z = 4$ and $\frac{x+3}{1} = \frac{y-6}{2}; z = 7$, leaving your answer in exact form.

(9)(i) Show the lines $\frac{x-1}{2} = \frac{y+3}{5} = \frac{z-2}{3}$ and $\frac{x}{1} = \frac{y-4}{2} = \frac{z+1}{2}$ are skew.

(ii) Find the shortest distance between the lines and identify the points on the lines at which this shortest distance occurs.

(10) Given that $A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $B = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$, $C = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, $D = \begin{pmatrix} p \\ 4 \\ -4 \end{pmatrix}$

(i) Write down the equations of the lines AB and CD (both extended)

(ii) Find $\overrightarrow{AB} \times \overrightarrow{CD}$

(iii) For what value of p are the lines AB and CD parallel? (2 methods)

(11) Find the plane containing the points
$(2, -1,4)$, $(-3,4,2)$ and $(1,0,5)$, in Cartesian form

(12)Find the reflection of the line $\frac{x-2}{3} = \frac{y+4}{1}; z = 3$ in the plane $y = 4$