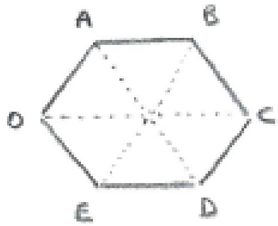
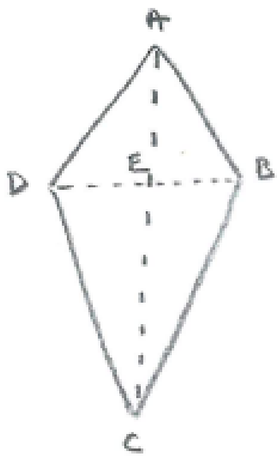
Vectors - Exercises (5 pages; 7/10/18)

(1) Referring to the regular hexagon below, if $\overrightarrow{OA} = \underline{a}$ & $\overrightarrow{OB} = \underline{b}$, find $\overrightarrow{OC}$, $\overrightarrow{BC}$ & $\overrightarrow{OD}$ in terms of $\underline{a}$ and $\underline{b}$

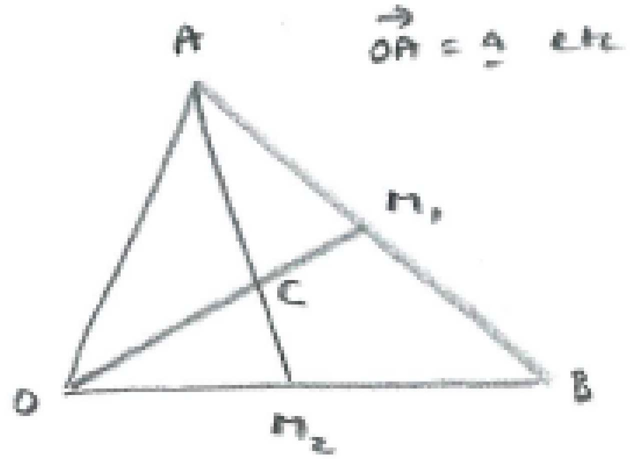


(2) [AEA, June 2009, Q7(d)]

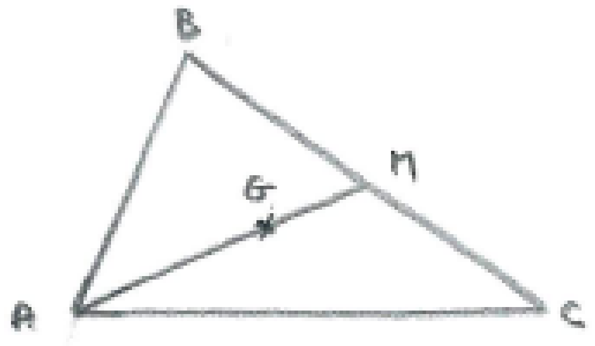
In the diagram below, ABCD is a kite. Find $\overrightarrow{OD}$ if $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 4/3 \\ 7 \end{pmatrix}$,
$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 4/3 \\ 2 \end{pmatrix}$ & $\overrightarrow{OC} = \begin{pmatrix} 6 \\ 16/3 \\ 2 \end{pmatrix}$



(3) Prove that the centre of mass of a triangular lamina lies 2/3 of the way along any of the medians.



(4) Given that the centre of mass of a triangular lamina lies 2/3 of the way along any of the medians, prove that it has position vector $\frac{1}{3}(\underline{a} + \underline{b} + \underline{c})$.



(5) Show that if $|\underline{a} - \underline{b}| = |\underline{a} + \underline{b}|$, then $\underline{a}$ & $\underline{b}$ are perpendicular.

(6) Find a vector equation of the line that passes through the point (1,2) and is perpendicular to the line $\underline{r} = \binom{3}{4} + \lambda\binom{4}{-1}$

(7) Use vectors to prove that the mid-points of the sides of any quadrilateral form the vertices of a parallelogram.

(8) Given that $A = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $B = \begin{pmatrix}-4\\3\\1\end{pmatrix}$, $C = \begin{pmatrix}0\\1\\2\end{pmatrix}$, $D = \begin{pmatrix}p\\4\\-4\end{pmatrix}$

(i) Write down the equations of the lines AB and CD (both extended)

(ii) Find $\overrightarrow{AB} \times \overrightarrow{CD}$

(iii) For what value of p are the lines AB and CD parallel? (2 methods)

(9) Find the angle between adjacent sloping faces of a right square-based pyramid, where the faces are equilateral triangles (as shown in Figure 1).

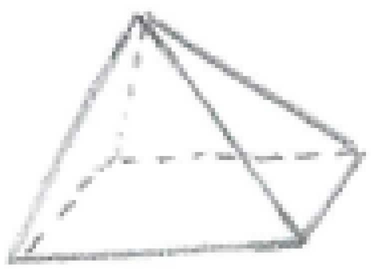
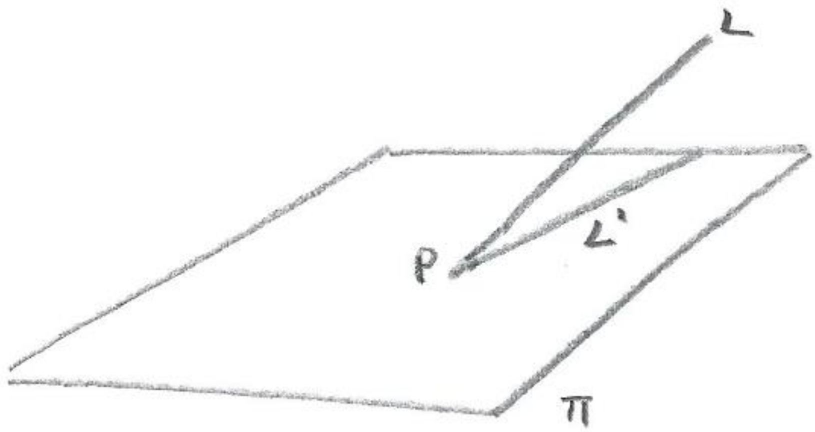


Figure 1

(10) Given the plane $\Pi: 3x + 2y - z = 6$ and the line

$L: \underline{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, let L' be the projection of L onto Π



(i) Find the point of intersection (P) of Π & L

(ii) Find the angle between Π & L

(iii) Find a vector that is parallel to Π and perpendicular to L

(iv) Find a vector equation for L'

(v) Find the angle between L and L'

(11) Given that $\boldsymbol{a}$, $\boldsymbol{b}$ & $\boldsymbol{c}$ are linearly independent vectors, establish whether the vectors $\boldsymbol{a}+\boldsymbol{b}$, $\boldsymbol{a}-\boldsymbol{c}$ & $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}$ are linearly independent.

(12) Are the vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$ & $\begin{pmatrix}0\\0\\0\end{pmatrix}$ linearly independent?

(13) Find the cartesian form of the plane

$$\underline{r} = \begin{pmatrix}0\\-2\\-1\end{pmatrix} + s\begin{pmatrix}1\\4\\4\end{pmatrix} + t\begin{pmatrix}2\\3\\1\end{pmatrix}$$

(14)(i) Find the intersection of the line $\underline{r} = \underline{a} + t\underline{b}$ and the plane $\underline{r}.\underline{n} = d$

(ii) Find the intersection of the line $\underline{r} = \begin{pmatrix}0\\1\\0\end{pmatrix} + t\begin{pmatrix}1\\2\\0\end{pmatrix}$ and the plane $\underline{r}.\begin{pmatrix}1\\-1\\0\end{pmatrix} = -2$