Useful Results - Applied (3 pages; 7/11/18)

Mechanics

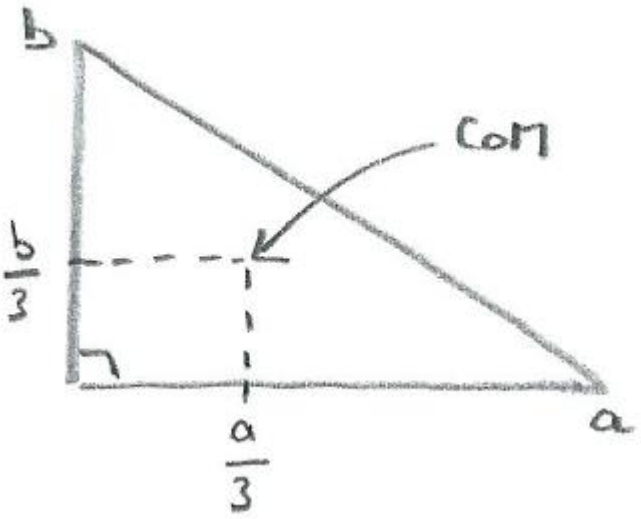
(1) Centre of Mass

Triangular lamina

(a) average of coordinates of vertices

(b) $\frac{2}{3}$ along median from vertex

(c) right-angled triangular lamina: see diagram below



Solid cone or pyramid of height h

$\frac{h}{4}$ from base (on line of symmetry)

Hollow cone or pyramid of height h

$\frac{h}{3}$ from base (on line of symmetry)

Sector of circle (radius r, angle 2θ at centre)

$\frac{2rsin\theta}{3\theta}$ [As $\theta \to 0$, $\frac{2rsin\theta}{3\theta} \to \frac{2r}{3}$ (as $\frac{sin\theta}{\theta} \to 1$) ; as $\theta \to \frac{\pi}{2}$, CoM moves nearer the centre, and $\frac{sin\theta}{\theta}$ reduces]

Arc of circle (radius r, angle 2θ at centre)

$\frac{rsin\theta}{\theta}$ [As $\theta \to 0, \frac{rsin\theta}{\theta} \to r$; as $\theta \to \frac{\pi}{2}$, CoM moves nearer the centre]

(2) Projectiles

Cartesian equation: $y = xtan\theta - \frac{gx^2}{2u^2cos^2\theta}$

(3) SHM

$\ddot{x} = -\omega^2 x$

$x = asin(\omega t + \epsilon)$

$v^2 = \omega^2(a^2 - x^2)$

(4) Rigid bodies

Moment of inertia, $I = \sum m_i {r_i}^2$

KE $= \frac{1}{2} I\omega^2$

Angular momentum $= I\omega$

Total moments of forces , $C = I\ddot{\theta}$

Work done $= \int C \, d\theta$

Statistics

(1) Normal probabilities

sd	prob. (1 tail)
1	16%
1.645	5%
1.96	2.5%
2.326	1%
2.576	0.5%

(2) de Morgan's Laws

P[(A∪B)'] = P[A'∩B']

P[(A∩B)'] = P[A'∪B']

(3) Probability Generating Functions

If $X_1, X_2, \ldots$ & N are independent random variables, where the X_i have pgf $G_X(s)$, then

(i) $S_N = X_1 + X_2 + \cdots + X_n$ has pgf $G_{S_N}(s) = G_N\big(G_X(s)\big)$

(ii) $E(S_N) = E(N)E(X)$

(iii) $Var(S_N) = E(N)Var(X) + Var(N)[E(X)]^2$