Turning points (1 page; 24/10/18)

See also: "Points of inflexion "

(1) The turning point of a quadratic is midway between its roots.

(2) Note that a turning point can occur when $\frac{d^2y}{dx^2} = 0$ (eg $y = x^4$), and the pattern of $\frac{dy}{dx}$ about the point may need to be examined, instead of the usual test of whether $\frac{d^2y}{dx^2}$ is positive or negative.

(3) A necessary and sufficient condition for a turning point is that the first non-vanishing derivative must be even (with order ≥ 2). The sign of this derivative then determines whether it is a maximum or minimum. Thus, in the case of $y = x^4$ at $x = 0$,

$\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} = 0$, $\frac{d^3y}{dx^3} = 0$ & $\frac{d^4y}{dx^4} = 24$

(4) Note that a maximum/minimum can occur without $\frac{dy}{dx} = 0$, if the domain of the function is limited, and the greatest/lowest value occurs at the boundary.

(5) A polynomial function of the form $(x - a)^{2m}g(x)$, where $m > 0$, has a turning point at $(a, 0)$.