Trigonometry Q8 (30/6/23)

Show that each of (i)-(vi) is true, by two methods:

(a) using the results (A)-(E) below
(b) from graphs

(i) $sin(\theta + 180) = -sin\theta$
(ii) $cos(180 - \theta) = cos\ (180 + \theta)$
(iii) $cos(90 - \theta) = -cos\ (90 + \theta)$
(iv) $sin(\theta - 180) = cos\ (\theta + 90)$
(v) $sin(\theta + 90) = cos\theta$
(vi) $\sin(360 - \theta)$

(A) $sin(-\theta) = -sin\theta$
(B) $sin(360 + \theta) = sin\theta$
(C) $sin(180 - \theta) = sin\theta$
(D) $sin\theta = cos\ (90 - \theta)$
(E) $cos(-\theta) = cos\theta$

Solution

(i)(a) $sin(\theta + 180) = sin(\theta + 180 - 360) = sin(\theta - 180)$

$= -sin(180 - \theta) = -sin\theta$

(b) Starting with the graph of $y = sin\theta$, $y = sin(\theta + 180)$ is obtained by a translation of 180° to the left, and this can be seen to be the graph of $y = -sin\theta$.

(ii)(a) $cos(180 - \theta) = \cos(\theta - 180) = \cos(\theta - 180 + 360)$

$= cos\ (180 + \theta)$

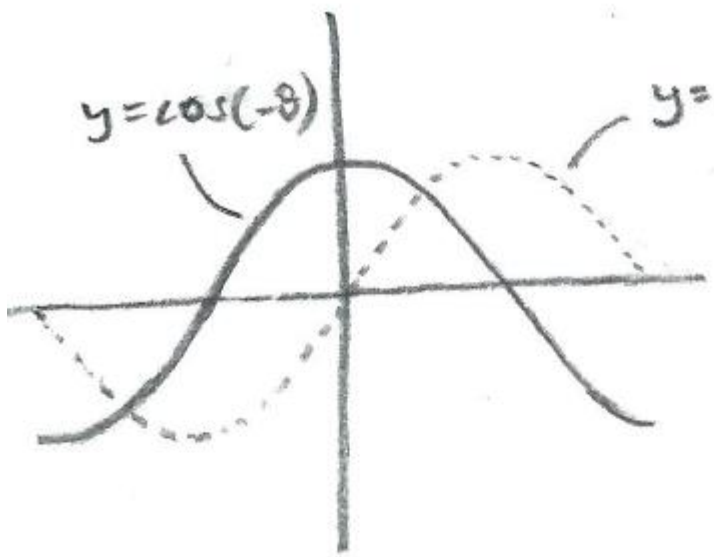
(b) Starting with the graph of $y = cos\theta$, $y = cos(\theta + 180)$ is obtained by a translation of 180° to the left, and this can be seen to have symmetry about the y-axis, so that replacing θ by $-\theta$ has no effect; ie $cos(\theta + 180) = cos(-\theta + 180) = cos\ (180 - \theta)$

(iii)(a) $cos(90 - \theta) = sin\theta = -\,sin(-\theta) = -cos\ (90 - [-\theta])$

$= -cos\ (90 + \theta)$

(b) The graph of $y = cos(90 - \theta)$ can be obtained from $y = cos\theta$ by a reflection in the y-axis (having no effect), to give $y = \cos\ (-\theta)$, followed by a translation of 90° to the right (replacing θ with $\theta - 90$) , to give $y = \cos\big(-(\theta - 90)\big)$

$= cos(90 - \theta)$ (see diagram below).



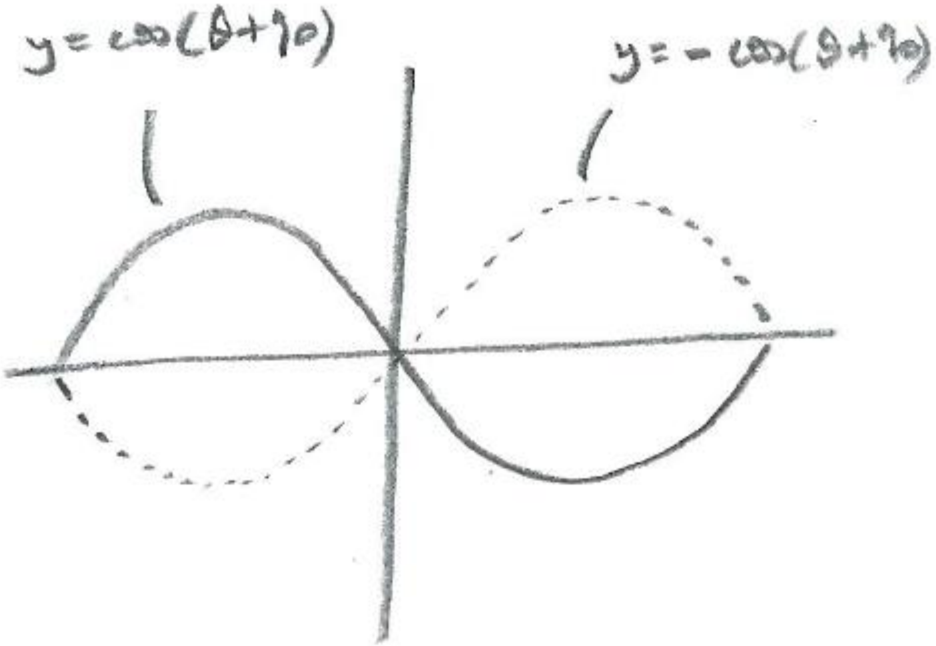
Then the graph of $y = -cos\ (90 + \theta)$ can be obtained from

$y = cos\theta$ by a translation of $90°$ to the left, to give

$y = cos(\theta + 90)$, followed by a reflection in the x-axis ,

to give $y = -\ cos(\theta + 90) = -cos\ (90 + \theta)$

(see diagram below).



And the graphs of $y = \cos\big(-(\theta - 90)\big)$ and $y = -\,cos(\theta + 90)$

are seen to be the same from these diagrams.

(iv)(a) $sin(\theta - 180) = sin(\theta - 180 + 360) = sin\ (\theta + 180)$

$= \cos(90 - [\theta + 180]) = \cos(-\theta - 90) = \cos\ (-[-\theta - 90])$

$= cos\ (\theta + 90)$

(b) The graph of $y = sin(\theta - 180)$ can be obtained from

$y = sin\theta$ by a translation of 180° to the right, whilst the graph of

$y = cos(\theta + 90)$ can be obtained from $y = cos\theta$ by a translation

of 90° to the left. The resulting graphs can be seen to be the same.

(v)(a) $sin(\theta + 90) = cos\ (90 - [\ \theta + 90]) = cos(-\theta) = cos\theta$

(b) The graph of $y = sin(\theta + 90)$ can be obtained from

$y = sin\theta$ by a translation of 90° to the left, which gives the graph

of $y = cos\theta$.

(vi)(a) $\sin(360 - \theta) = \sin(-\theta) = -sin\theta$

(b) Starting at 360 and moving θ to the left gives the same absolute value for $sin\theta$ as starting from 0 and moving θ to the right, with just a change of sign.