

Trigonometry Q7 (30/6/23)

Given that $\cos^5 \theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos \theta)$ and

$$\cos^6 \theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10),$$

find expressions for $\sin^5 \theta$ and $\sin^6 \theta$

Solution

$$\begin{aligned}
\sin^5 \theta &= \cos^5 \left(\frac{\pi}{2} - \theta \right) \\
&= \frac{1}{16} (\cos [5 \left(\frac{\pi}{2} - \theta \right)] + 5 \cos [3 \left(\frac{\pi}{2} - \theta \right)] + 10 \cos \left(\frac{\pi}{2} - \theta \right)) \\
&= \frac{1}{16} (\cos \left[\frac{\pi}{2} - 5\theta \right] + 5 \cos \left[-\frac{\pi}{2} - 3\theta \right] + 10 \sin \theta) \\
&= \frac{1}{16} (\sin 5\theta + 5 \cos \left(\frac{\pi}{2} + 3\theta \right) + 10 \sin \theta) \\
&= \frac{1}{16} (\sin 5\theta + 5 \cos \left(\frac{\pi}{2} - [-3\theta] \right) + 10 \sin \theta) \\
&= \frac{1}{16} (\sin 5\theta + 5 \sin (-3\theta) + 10 \sin \theta) \\
&= \frac{1}{16} (\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)
\end{aligned}$$

$$\begin{aligned}
\text{And } \sin^6 \theta &= \cos^6 \left(\frac{\pi}{2} - \theta \right) \\
&= \frac{1}{32} (\cos [6 \left(\frac{\pi}{2} - \theta \right)] + 6 \cos [4 \left(\frac{\pi}{2} - \theta \right)] + 15 \cos [2 \left(\frac{\pi}{2} - \theta \right)] + 10) \\
&= \frac{1}{32} (\cos (\pi - 6\theta) + 6 \cos (-4\theta) + 15 \cos (\pi - 2\theta) + 10) \\
&= \frac{1}{32} (-\cos 6\theta + 6 \cos 4\theta - 15 \cos 2\theta + 10)
\end{aligned}$$