Trigonometry Q6 (30/6/23)

Solve $\sin(2\theta - \frac{\pi}{6}) = 0.5 \ \ (0 < \theta < 2\pi)$

Solution

Let $u = 2\theta - \frac{\pi}{6}$, so that $-\frac{\pi}{6} < u < 4\pi - \frac{\pi}{6}$

Then $sin\, u = 0.5 \Rightarrow u = \frac{\pi}{6}, \frac{\pi}{6} + 2\pi$ and $\pi - \frac{\pi}{6}, \pi - \frac{\pi}{6} + 2\pi$

ie $u = \frac{\pi}{6}, \frac{13\pi}{6}, \frac{5\pi}{6}$ & $\frac{17\pi}{6}$ or $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ & $\frac{17\pi}{6}$

so that $\theta = \frac{1}{2}\left(u + \frac{\pi}{6}\right) = \frac{2\pi}{12}, \frac{6\pi}{12}, \frac{14\pi}{12}$ & $\frac{18\pi}{12}$

ie $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}$ & $\frac{3\pi}{2}$