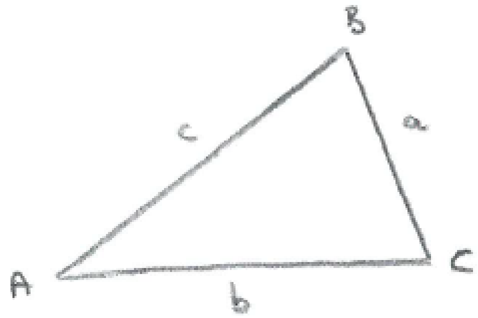
Trigonometry Q10 (30/6/23)

Denote the sides of a triangle by a, b & c, and the angles (opposite these sides respectively) by A, B & C, as in the diagram below.



(i) What combinations of sides and angles will always enable the other sides and angles to be determined uniquely (ie any two triangles thus created will be congruent, so that a reflection in the plane of the paper is allowed)? What combination gives rise to two possibilities in some cases?

(ii) When finding missing lengths and angles, what ambiguous situation can arise (apart from the 2 solutions in IV), and how can it be avoided?

Solution

(i)

(I) a, b & c known ⇒ unique solution

(II) A, B (and hence C) & a (eg) ⇒ unique solution

(III) a, b & C $(or\ b, c\ \&\ A\ etc)$ ⇒ unique solution

(IV) a, b, A $(or\ B)$ ⇒ 2 solutions in some cases (if A is acute, $a < b$ and $B \neq 90°$)

(ii) If we attempt to find an angle that is close to 90° using the Sine rule, then it will not be clear whether the correct angle is θ or $180 - \theta$.

To avoid this problem we can either use the Cosine rule, or apply the Sine rule only to angles that are clearly acute, and deduce the remaining angle by subtraction from 180°. Note that the angle opposite the longest side is the only one that can be obtuse.