

Statistics - Exercises (3 pages; 21/2/17)

Binomial distribution

(1) If $X \sim B(n, p)$, prove that:

(i) $E(X) = np$ (ii) $Var(X) = np(1 - p)$

Normal distribution

(2) Suppose that the heights (in cm) of adult males in the UK are distributed $N(174, 49)$.

(i) Assuming that there are 2.5 cm to an inch, what proportion of adult males in the UK are over 6 ft? Give your answer to 1dp.

(ii) In another country, the heights of adult males are distributed Normally, such that 10% are over 6 ft and 5% are under 5ft. Find the mean and variance of the distribution. Give your answers to 1dp.

(3) Show that 1 standard deviation to either side of the mean of the Normal distribution occurs at the point of inflexion of the Normal curve.

Rank Correlation

(4) Show that the formula $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$ can be written as $r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$

when the data items are ranks.

[In other words, instead of using the formula for Spearman's coefficient, it is theoretically possible to use the standard formula for r .]

Venn Diagrams

(5) The events A and B are independent and are such that $P(A) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$

Find:

(i) $P(B)$

(ii) $P(A' \cup B')$

(iii) $P(B' \cap A)$

(iv) $P(B'|A)$

Probability generating functions

(6) Given that X_1, X_2, \dots, X_N & N are independent random variables, where the X_i are all distributed as X , and that

$$S_N = X_1 + X_2 + \dots + X_N,$$

prove that $Var(S_N) = E(N)Var(X) + Var(N)[E(X)]^2$

The following results may be used:

(A) $E(X) = G'_X(1)$

(B) $VarX = G''_X(1) + G'_X(1) - [G'_X(1)]^2$

(C) $G_{S_N}(s) = G_N(G_X(s))$

(D) $E(S_N) = E(N)E(X)$

(7) ['Poisson hen'] A hen lays N eggs, where $N \sim P_o(\lambda)$, and each egg has probability p of hatching. Using any results about probability generating functions, show that the total number of eggs that hatch $\sim P_o(\lambda p)$.