Series – Q5 [Practice/E] (3/3/23)

By considering $\sum_{r=1}^{n}(r+1)^3 - \sum_{r=1}^{n} r^3$ (*), show that

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ [assuming that $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$]

Solution

$(*) = 3(\sum_{r=1}^{n} r^2) + 3(\sum_{r=1}^{n} r) + n$

Also, $(*) = (n+1)^3 - 1$

Hence $3(\sum_{r=1}^{n} r^2) = (n+1)^3 - 1 - \frac{3}{2}n(n+1) - n$

$\Rightarrow 6(\sum_{r=1}^{n} r^2) = (n+1)\{2(n^2 + 2n + 1) - 2 - 3n\}$

$= (n+1)(2n^2 + n)$

$= n(n+1)(2n+1)$

$\Rightarrow \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$