# **Series – Q4: Method of Differences [5 marks]** (20/6/21)

## Exam Boards

OCR : Pure Core (Year 2)

MEI:  Core Pure (Year 1)

AQA: Pure (Year 1)

Edx: Core Pure (Year 2)

Given that $\frac{1}{r+1}-\frac{2}{r+2}+\frac{1}{r+3}=\frac{2}{(r+1)(r+2)(r+3)}$, use the method of differences to show that $\sum_{r=1}^{n}\frac{1}{(r+1)(r+2)(r+3)}=\frac{n(n+5)}{12(n+2)(n+3)}$

[5 marks]

## Solution

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \frac{1}{2}\{\sum_{r=1}^{n} \frac{1}{r+1} - \sum_{r=1}^{n} \frac{2}{r+2} + \sum_{r=1}^{n} \frac{1}{r+3}\}$$

$$= \frac{1}{2}\{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{n-2} + \frac{1}{n-1} + \frac{1}{n} + \frac{1}{n+1}$$

$$-2(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{n-1} + \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2})$$

$$+\frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3})\}$$ [2 marks]

As the highlighted items cancel: [1 mark]

$$= \frac{1}{2}\{\frac{1}{2} - \frac{1}{3} - \frac{1}{n+2} + \frac{1}{n+3}\}$$ [1 mark]

$$= \frac{(n+2)(n+3) - 6(n+3) + 6(n+2)}{12(n+2)(n+3)}$$

$$= \frac{n^2 + 5n}{12(n+2)(n+3)}$$

$$= \frac{n(n+5)}{12(n+2)(n+3)}$$ [1 mark]