

## **Series – Q1: Method of Differences [4 marks] (20/6/21)**

### **Exam Boards**

OCR : Pure Core (Year 2)

MEI: Core Pure (Year 1)

AQA: Pure (Year 1)

Edx: Core Pure (Year 2)

Given that  $\frac{2r+1}{r^2(r+1)^2} = \frac{1}{r^2} - \frac{1}{(r+1)^2}$ , use the method of differences to find  $\sum_{r=1}^n \frac{2r+1}{r^2(r+1)^2}$  [4 marks]

## Solution

$$\begin{aligned}\sum_{r=1}^n \frac{2r+1}{r^2(r+1)^2} &= \sum_{r=1}^n \frac{1}{r^2} - \frac{1}{(r+1)^2} \\ &= \left( \frac{1}{1^2} + \left[ \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{(n-1)^2} + \frac{1}{n^2} \right] \right) \\ &\quad - \left( \left[ \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots + \frac{1}{n^2} \right] + \frac{1}{(n+1)^2} \right) \quad [1 \text{ mark}]\end{aligned}$$

As the terms in square brackets cancel: [1 mark]

$$\begin{aligned}&= \frac{1}{1^2} - \frac{1}{(n+1)^2} = \frac{(n+1)^2 - 1}{(n+1)^2} \quad [1 \text{ mark}] \\ &= \frac{n^2 + 2n}{(n+1)^2} \quad [1 \text{ mark}] \\ &= \frac{n(n+2)}{(n+1)^2}\end{aligned}$$