Series Overview (28/6/23)

General

Q5 [Practice/E]

By considering $\sum_{r=1}^{n}(r+1)^3 - \sum_{r=1}^{n} r^3$ (*), show that

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ [assuming that $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$]

Q6 [Practice/H]

By considering $\sum_{r=1}^{n}(r+1)^4$, show that

$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ [The results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ can be assumed.]

Q7 [Practice/M]

Starting from $\sum_{r=1}^{n}(r+1)^2 = (\sum_{r=1}^{n} r^2) + 2(\sum_{r=1}^{n} r) + n$,

make the substitution $R = r + 1$,

to prove that $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

Method of Differences

Q1 [4 marks]

Given that $\frac{2r+1}{r^2(r+1)^2} = \frac{1}{r^2} - \frac{1}{(r+1)^2}$, use the method of differences to find $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$

Q2 [3 marks]

Given that $\frac{1}{2r} - \frac{1}{2(r+2)} = \frac{1}{r(r+2)}$, use the method of differences to find $\sum_{r=1}^{n} \frac{1}{r(r+2)}$ [3 marks]

Q3 [5 marks]

Given that $\frac{1}{2r-1} - \frac{2}{2r+1} + \frac{1}{2r+3} = \frac{8}{(2r-1)(2r+1)(2r+3)}$, use the method of differences to show that

$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)(2r+3)} = \frac{n(n+2)}{3(2n+1)(2n+3)}$ [5 marks]

Q4 [5 marks]

Given that $\frac{1}{r+1} - \frac{2}{r+2} + \frac{1}{r+3} = \frac{2}{(r+1)(r+2)(r+3)}$, use the method of differences to show that $\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \frac{n(n+5)}{12(n+2)(n+3)}$

[5 marks]