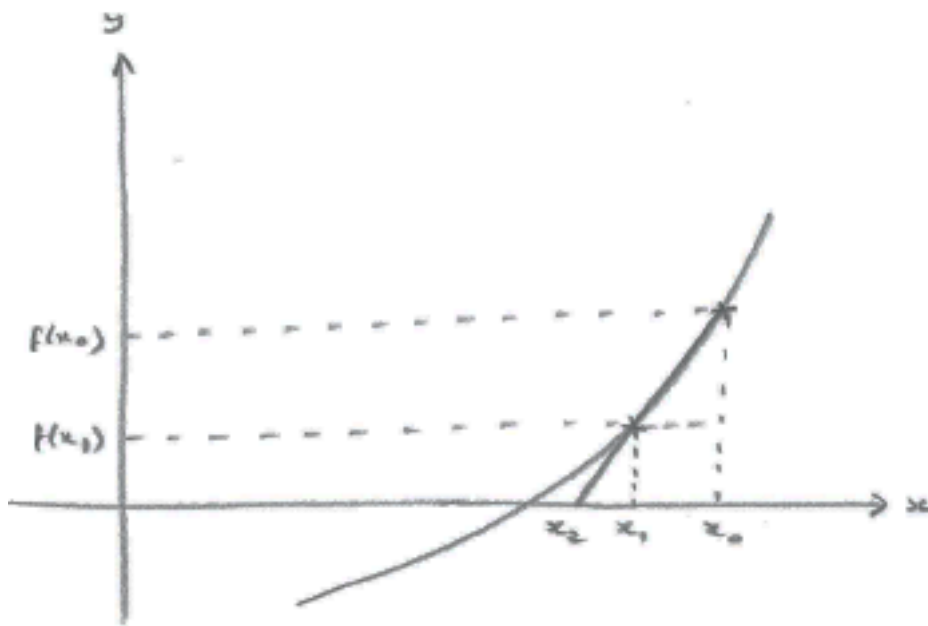


Numerical Solution of Equations - Secant method

(2 pages; 22/10/18)

(1) Referring to the diagram below, the Secant method formula is:

$$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)} \quad (\text{where } x_0 \text{ \& } x_1 \text{ are two initial estimates}).$$



Then $x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$ etc.

(2) The formula can be derived from the Newton-Raphson formula:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad \text{and} \quad f'(x_1) \approx \frac{f(x_0) - f(x_1)}{x_0 - x_1}$$

$$\text{so that } x_2 = x_1 - \frac{f(x_1)}{\left(\frac{f(x_0) - f(x_1)}{x_0 - x_1}\right)} = x_1 - \frac{f(x_1)(x_0 - x_1)}{f(x_0) - f(x_1)}$$

$$= \frac{x_1 f(x_0) - x_1 f(x_1) - x_0 f(x_1) + x_1 f(x_1)}{f(x_0) - f(x_1)}$$

$$= \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)}$$

(3) The formula can also be derived by linear interpolation:

Consider the situation where x_2 lies between x_1 and x_0 . Then x_2 is a weighted average of x_1 and x_0 :

$$x_2 = \left(\frac{f(x_0) - f(x_2)}{f(x_0) - f(x_1)} \right) x_1 + \left(\frac{f(x_2) - f(x_1)}{f(x_0) - f(x_1)} \right) x_0$$

The formula will be valid for all positions of x_2 , and since

$$f(x_2) = 0, \quad x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)}$$

(4) Example: A spreadsheet can be used to find the root of

$$x^3 - x - 1 = 0, \text{ with } x_0 = 2 \text{ and } x_1 = 1.5$$

	A	B	C	D	E	F
2	x(r)	f(x(r))				
3						
4	2	5.000000000				
5	1.5	0.875000000				
6	1.393939	0.314578290				
7	1.334405	0.041685543				
8	1.325311	0.002529955				
9	1.324723	0.000022667				
10	1.324718	0.000000013				

A6 : f_x $=+(A4*B5-A5*B4)/(B5-B4)$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$