STEP Problems - Trigonometry (1 page; 7/9/18)

(1) Solve $sin\theta = cos4\theta$ for $0 < \theta < \pi$

(2) Given that $cos^5\theta = \frac{1}{16}(cos5\theta + 5cos3\theta + 10cos\theta)$ and

$cos^6\theta = \frac{1}{32}(cos6\theta + 6cos4\theta + 15cos2\theta + 10)$,

find expressions for $sin^5\theta$ and $sin^6\theta$

(3) Simplify $\sqrt{2(1 - cos\theta)}$ and $\sqrt{2(1 + cos\theta)}$

(4) Assuming that $sin^2\theta + cos^2\theta = 1$, but without using any compound angle results, show that $sin\theta cos\theta \leq \frac{1}{2}$

(5) Show that $\arctan\left(\frac{1+a}{\sqrt{1-a^2}}\right) - \arctan\left(\frac{a}{\sqrt{1-a^2}}\right) = \arctan\left(\frac{\sqrt{1-a}}{\sqrt{1+a}}\right)$

(6) What is the period of $2\,sin\left(3x + \frac{\pi}{4}\right) + 3cos(\frac{2x}{3} - \frac{\pi}{3})$?