STEP/Polynomials Q2 (26/6/23)

Factorise (a) $x^{3}-y^{3}$ (b) $x^{3}+y^{3}$

Solution
(a) Let $f(x)=x^{3}-y^{3}$

By the Factor theorem (treating $f(x)$ as a cubic in $x$ ), since

$$
\begin{aligned}
& f(y)=0,(x-y) \text { is a factor of } x^{3}-y^{3}, \text { leading to } \\
& x^{3}-y^{3}=(x-y)\left(x^{2}+x y+y^{2}\right)
\end{aligned}
$$

(b) Similarly, $(x+y)$ is a factor of $x^{3}+y^{3}$, leading to $x^{3}+y^{3}=(x+y)\left(x^{2}-x y+y^{2}\right)$

