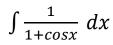
STEP/Integration Q9 (21/6/23)



Solution

$$\int \frac{1}{1+\cos x} dx = \int \frac{1-\cos x}{1-\cos^2 x} dx = \int \frac{1-\cos x}{\sin^2 x} dx$$
$$= \int \csc^2 x \, dx - \int \frac{\cos x}{\sin^2 x} dx$$

Now, as
$$\frac{d}{dx}tanx = sec^2 x$$
, we can expect $\frac{d}{dx}cotx = a.cosec^2 x$,

where
$$a = 1$$
 or -1 .

To investigate this, $\frac{d}{dx} (tanx)^{-1} = -(tanx)^{-2}sec^2x = -cosec^2x$

For the 2nd integral, as the integral of the numerator *cosx* features simply in the rest of the integrand (ie $\frac{1}{sin^2x}$ can be written as $\frac{1}{u^2}$, where u = sinx, and $\frac{1}{u^2}$ can easily be integrated), u = sinx leads to $\int \frac{cosx}{sin^2x} dx = -\frac{1}{sinx} = -cosecx$ So $\int \frac{1}{1+cosx} dx = -cotx + cosecx + c$